POLYTROPIC-COEFFICIENT FUNCTION (PCF) VS. 
POLYTROPIC-EXPONENT FUNCTION (PEF)*

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In fluid treatments of plasma-wall-transition (PWT) problems, the system of fluid equations of a given particle species is often1-3 closed by means of an equation of the form

$$\frac{Dp}{Dt} = \left( \gamma \frac{n}{\rho} \right) \frac{Dn}{Dt} \quad \text{("local polytropic law")},$$

where \(n(\vec{x},t)\) and \(p(\vec{x},t)\) are the density and pressure, \(\gamma(\vec{x},t)\) is called the “polytropic-coefficient function (PCF)”3, and

$$\frac{D}{Dt} = \partial_t + \vec{u} \cdot \vec{\nabla}$$

is the Lagrangian time derivative for a “streaming” point observer, i.e., an observer moving along with a fluid “particle” in the flow-velocity field \(\vec{u}(\vec{x},t)\).

In the classical thermodynamics of a uniform macroscopic system with volume \(V\) and pressure \(p\), a polytropic process is defined by the “polytropic equation” \(p V^{\gamma_{pe}} = \text{const}\) with the “polytropic exponent” \(\gamma_{pe}\) constant over some finite range of states. To apply this concept to the generally non-uniform and time-dependent conditions of flow problems, we replace the macroscopic system of volume \(V\) with an infinitesimal “streaming” fluid element of volume \(\delta V\) traveling along its trajectory during an infinitesimal time interval \(Dt\) through all points \((\vec{x}',t')\) varying continuously from the initial point \((\vec{x}_i = \vec{x} - \vec{u} Dt/2, t_i = t - Dt/2)\) via the central point \((\vec{x},t)\) to the final point \((\vec{x}_f = \vec{x} + \vec{u} Dt/2, t_f = t + Dt/2)\), where \(\vec{u} = \vec{u}(\vec{x},t)\). For this infinitesimal thermodynamic system moving an infinitesimal distance we can define a “local polytropic equation” of the form \(p(\delta V')^{\gamma(\vec{x},t)} = \text{const}\), the polytropic exponent of which is given by the “polytropic-exponent function” (PEF) \(\gamma_{pe}(\vec{x},t)\) taken at the central point and, hence, constant during the infinitesimal process considered.

In the present contribution we will (i) show that the PCF and the PEF are related by

$$\frac{\gamma}{\gamma_{pe}} = 1 - S_n \left( n \vec{\nabla} \cdot \vec{u} \right),$$

where \(S_n(\vec{x},t)\) is the particle-source term on the rhs of the continuity equation, and (ii) discuss this relationship for several special cases such as isothermal and incompressible flows.


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