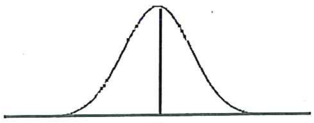
business maths foundations

6.6

The Normal Distribution

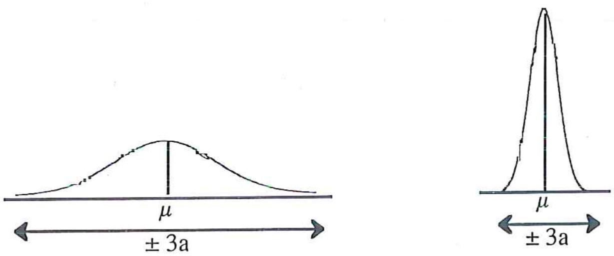
1. Introduction

The **Normal Curve** or **Normal Distribution** is very important in Statistics.



The highest point (the most frequent result) is at the centre. This is the **MEAN** (μ). There is an equal and exactly similar curve on each side of the centre so that half the values are above the mean and half below the mean. It is bell-shaped and symmetrical, so the frequency at 2 units above the mean is the same as the frequency at 2 units below the mean. The Median and Mode coincide with the Mean and the frequencies diminish as you move away from the mean. The **area** under the curve indicates the total frequencies. The total area under the curve = 1. The Mean and Standard deviation determine the shape

eg



68% of the data lies with $\pm 1\sigma$ of the mean, 95% of the data lies within $\pm 2\sigma$ of the mean and 99.7% lies within $\pm 3\sigma$ of the mean.

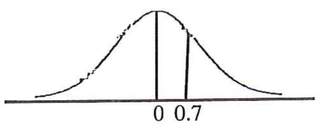
2. Normal Distribution

Worked example

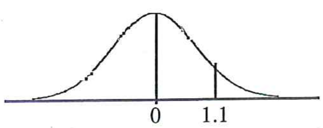
A Normal Distribution with mean 0 and Standard deviation 1 is written $N(0,1)$ i.e.: $N(\mu, \sigma^2)$

Find the probability (a) $Z < 0.7$ (b) $Z > 1.1$ (c) $0.7 < Z < 1.1$

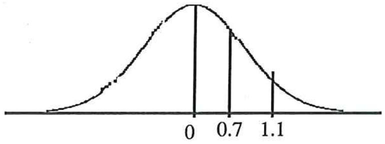
a) $P(Z < 0.7) = 0.5 + \phi(0.7)$
$= 0.5 + 0.2580$ (from tables)
$= 0.7580$



b) $P(Z > 1.1) = 0.5 - \phi(1.1)$
$= 0.5 - 0.3643$ (from tables)
$= 0.1357$



c) $P(0.7 < Z < 1.1) = 1 - (0.7580 + 0.357)$
$= 1 - 0.8937$
$= 0.1063$



3. The Standard Normal Curve

The standard Normal distribution is $Z \sim N(0, 1^2)$.

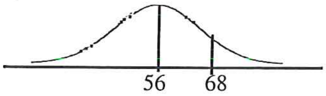
We can transform any Normal Distribution $Z \sim N(\mu, \sigma^2)$ to the Standard Normal distribution using: $Z = \frac{X-\mu}{\sigma}$

Worked Example

$Y \sim N(56, 10^2)$ i.e. the mean is 56 and the standard deviation is 10.

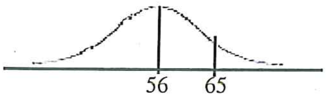
Find: a) $P(Y > 68)$ b) $P(56 < Y < 65$ c) $P(42 < Y < 52)$

a) $P(Y > 68)$
$$= P\left(Z > \frac{68-56}{10}\right)$$
$$= P(Z > 1.2)$$
$$= 0.5 - 0.3849 \quad \text{(from tables)}$$



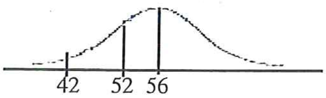
$$= 0.1151$$

b) $P(56 < Y < 65)$
$$= P\left(\frac{56-56}{10} < Z < \frac{52-56}{10}\right)$$
$$= P(0 < Z <> 0.9)$$



$$= 0.3159 \quad \text{(from tables)}$$

c) $P(42 < Y < 52)$
$$= P\left(\frac{42-56}{10}\right) < Z < \frac{52-56}{10}$$
$$= P(-1.4 < Z < -0.4)$$



$$= \phi(-1.4) - \phi(-0.4)$$
$$= 0.4192 - 0.1555 \quad \text{(from tables)}$$
$$= 0.2637$$

Exercises

1. $Z \sim N(0, 1^2)$, find:

a) $P(Z < 1.7)$ b) $P(Z < -0.7)$ c) $P(Z > 2.2)$
d) $P(-1.5 < Z < 1.5)$ e) $P(2.1 < Z < 2.9)$ f) $P(-2.1 < Z < 0.8)$

2. $Y \sim N(28, 3^2)$, find:

a) $P(Y > 32)$ b) $P(28 < Y < 35)$ c) $P(25 < Y < 33)$

Solutions

1. a) 0.9554 b) 0.2420 c) 0.0139
d) 0.8664 e) 0.0160 f) 0.7702

2. a) 0.0918 b) 0.4901 c) 0.7938

THE NORMAL PROBABILITY INTEGRAL

$z = \frac{x-\mu}{\sigma}$	0	1	2	3	4	5	6	7	8	9
0	0000	0040	0080	0120	0160	0199	0239	0279	0319	0359
.1	0398	0438	0478	0517	0577	0596	0636	0675	0714	0753
.2	0793	0832	0871	0909	0948	0987	1026	1064	1103	1141
.3	1179	1217	1255	1293	1331	1368	1406	1443	1480	1517
.4	1555	1591	1628	1664	1700	1736	1772	1808	1844	1879
.5	1915	1950	1985	2019	2054	2088	2123	2157	2190	2224
.6	2257	2291	2324	2357	2389	2422	2454	2486	2517	2549
.7	2580	2611	2642	2673	2703	2734	2764	2794	2822	2852
.8	2881	2910	2939	2967	2995	3023	3051	3078	3106	3133
.9	3159	3186	3212	3238	3264	3289	3315	3340	3365	3389
1.0	3413	3438	3461	3485	3508	3531	3554	3577	3599	3621
1.1	3643	3665	3686	3708	3729	3749	3770	3790	3810	3830
1.2	3849	3869	3888	3907	3925	3944	3962	3980	3997	4015
1.3	4032	4049	4066	4082	4099	4115	4131	4147	4162	4177
1.4	4192	4207	4222	4236	4251	4265	4279	4292	4306	4319
1.5	4332	4345	4357	4370	4382	4394	4406	4418	4429	4441