## mathcentre

## The Mean

## Introduction

This page will show how to calculate the mean from a frequency table and how to obtain an estimate of the mean from a grouped frequency table.

## Frequency Tables

When raw data is organised it can be helpful to display it in the form of a table showing the frequency $(f)$ with which each data item ( $x$ ) occurs.
Such a table is called a frequency table. However, when a larger range of data is involved it may be beneficial to first break the data down into small groups, in which case, the resulting table is referred to as a grouped frequency table.

An example of each type of table is shown in Figures 1 and 2.
The number of telephone calls received by a company switchboard over 5 minute intervals

| Number of Calls $(x)$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency $(f)$ | 7 | 10 | 15 | 29 | 13 | 6 |

Fig. 1 An example of a frequency table
Test marks of 25 students

| Marks $(x)$ | $1-10$ | $11-20$ | $21-30$ | $31-40$ |
| :---: | :---: | :---: | :---: | :---: |
| Frequency | 1 | 9 | 8 | 7 |

Fig. 2 An example of a grouped frequency table

## Calculating the Mean from a Frequency Table

In Mean, Median and Mode the formula to calculate the mean $(\bar{x})$ was seen to be $\frac{\sum_{n} x}{n}$.

## Worked example

| $x$ | $x_{1}$ | $x_{2}$ | $x_{3}$ | $x_{4}$ | $x_{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 2 | 2 | 5 | 5 |

$\sum x=2+2+2+5+5=16$
giving $\bar{x}=\frac{16}{5}=3.2$

However, since the value 2 occurred 3 times and the value 5 occurred twice we may write this data in the form of a frequency table as | $x$ | 2 | 5 |
| :---: | :---: | :---: |
| f | 3 | 2 | and obtain the same result for $\sum x$

by doing $(2+2+2)+(5+5)=3 \times 2+2 \times 5=6+10=16$
Thus we see that when a particular value of $x$ occurs $f$ times the formula for the mean becomes
$\bar{x}=\frac{\sum f x}{n}$
This formula with now be applied to the data displayed in Fig. 1.

The table is displayed with an additional row for the product $f x$.

| Number of calls $(x)$ |  | 0 | 1 | 2 | 3 | 4 | 5 | Totals |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | :---: |
| Frequency | $(f)$ | 7 | 10 | 15 | 29 | 13 | 6 | 80 |
| Product | $(f x)$ | 0 | 10 | 30 | 87 | 52 | 30 | 209 |

$\bar{x}=\frac{\sum f x}{n}=\frac{209}{80}=2.6125$ calls.

## Estimating the Mean from a Grouped Frequency Table

Referring to the data displayed in Fig. 2 we notice that individual values of $x$ are no longer available for us to use in our calculation. We know that in the 1-10 group there is a single value, but we do not know what that value is precisely.
We therefore make an estimate of its value by considering it to be located at the midpoint of the interval i.e. $x=\frac{1+10}{2}=5.5$
We do the same in all other intervals, using the mid point to represent all the values of $x$ that occur in those intervals.
Sometimes, perhaps our estimate will be too big, sometimes too small, but overall we hope that these estimation errors will cancel out or become insignificant, leading to a good estimate of the mean value.

The following table shows the results of the calculation of this estimate.

| Marks $\quad(x)$ | $1-10$ | $11-20$ | $21-30$ | $31-40$ | Totals |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Frequency $(f)$ | 1 | 9 | 8 | 7 | 25 |
| Midpoint $(x)$ | 5.5 | 15.5 | 25.5 | 35.5 |  |
| Product $(f x)$ | 5.5 | 139.5 | 204 | 248.5 | 597.5 |

$\bar{x}=\frac{\sum f x}{n}=\frac{597.5}{25}=23.9$ calls.

## Exercises

1. Calculate the mean of the following:
a) The number of faulty components from a production line, over 30 days

| No. faulty | 0 1 2 3 4 <br> 5     <br> Frequency 9 12 5 3 | 0 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |

b) The number of people in each car passing a checkpoint

| No. people | 1 | 2 | 3 | 4 | 5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Frequency | 37 | 13 | 6 | 3 | 1 |

2. Calculate an estimate of the mean of the following:

a) | Height $(\mathrm{cm})$ of students | $164 \leq h<168$ | $168 \leq h<172$ | $172 \leq h<176$ | $176 \leq h<180$ |
| :--- | :---: | :---: | :---: | :---: |
| Frequency | 6 | 17 | 14 | 3 |

b) | Time (min) of clinical consultations | $0 \leq t<5$ | $5 \leq t<10$ | $10 \leq t<15$ | $15 \leq t<20$ |
| :--- | :---: | :---: | :---: | :---: |
| Frequency | 22 | 17 | 9 | 2 |

## Answers

1. a) 1.2
b) 1.63 ( 2 dp )
2. a) 171.4
b) 6.6
