Loughborough University

Week 6: More Integration, Trigonometric Functions

Solutions

1. Evaluate the following integral using partial fractions

   $\int \frac{x^2+4}{3x^3+4x^2-4x}, dx$

   Solution: $\int \frac{x^2+4}{3x^3+4x^2-4x}, dx = \int -\frac{1}{x} + \frac{\frac{1}{2}}{x+2} + \frac{\frac{5}{2}}{3x-2}, dx$

   $= -\ln|x| + \frac{1}{2}\ln|x+2| + \frac{5}{6}\ln|3x-2| + c$

   Again, as noted above, integrals that generate natural logarithms are very common in these problems so make sure you can do them. Also, you were able to correctly do the last integral right? The coefficient of $\frac{5}{6}$ is correct. Make sure that you do the substitution

2. Evaluate the following integral using partial fractions

   $\int \frac{x^2-29x+5}{(x-4)^2(x^2+3)}, dx$

   Solution: $-\frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| - \frac{1}{x-1} + c$.

3. Evaluate the following integral

   (a) Use the substitution $x = u^2, u > 0$, to show that

   $\int \frac{1}{x(2\sqrt{x}-1)}\, dx = \int \frac{1}{u^2(2u-1)} 2u du \int \frac{2}{u(2u-1)}\, du$

   (b) Hence show that $\int_1^9 \frac{1}{x(2\sqrt{x}-1)}\, dx = 2\ln(\frac{a}{b})$

   where a and b are to be determined.

   Solution: $\int_1^9 \frac{1}{x(2\sqrt{x}-1)}\, dx = 2[-\ln(u) + \ln(2u-1)]_1^9 = 2[\ln(\frac{2u-1}{y})]_1^9 = 2\ln(\frac{a}{b})$.

4. Convert each angle from radians to degrees, giving your answers to 1 decimal place:

   a) 2^c b) 0.5^c c) 3.1^c d) 1.43^c e) 8.7^c f) 0.742^c

   Solution:

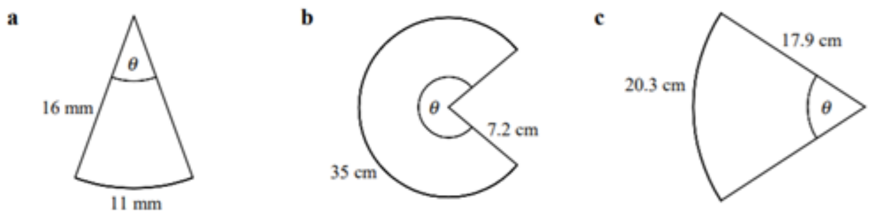
   a) $114.6°$ b) $28.6°$ c) $177.6°$ d) $81.9°$ e) $498.5°$ f) $42.5°$

5. Convert to radiants

   a) 120^o b) 135^o c) $450^o s$

   Solution: a) $\frac{2\pi}{3} = 2.09\ rads$ b) $\frac{3\pi}{4} = 2.36^c$ c) $\frac{5\pi}{2} = 7.85^c$

6. Using the formula $s = r\theta$, calculate the angle θ in each of the following circular sectors:



Solution: 0.6875^c, 4.861^c, 1.134^c

7. Sketch, over $0 < \theta < 2\pi$ the graph of $\sin 2\theta$. Mark the horizontal axis in radiants. Write down the period of $\sin\theta$.
   Solution: $\sin 2\theta$ has period π.