Week 5: Calculus

Solutions

1. What does the notation $\frac{dy}{dx}$ mean if you consider the graph of the function $y = x^2 + 2x - 1$?

   Solution: $\frac{dy}{dx}$ gives the gradient of the curve.

2. Find $\frac{dy}{dx}$ for each of these functions: (a) $y = x^2 + 2x$ (b) $y = 5x^3 - 1$

   Solution: (a) $\frac{dy}{dx} = 2x + 2$, (b) $\frac{dy}{dx} = 15x^2$

3. Calculate the gradient of the curve $y = x^2 + 2x - 1$ when $x = 0, x = 2$ and $x = -1$

   Solution: $\frac{dy}{dx} = 2x + 2$. When $x = 0$, $\frac{dy}{dx} = 2$; when $x = 2$, $\frac{dy}{dx} = 6$; when $x = -1$, $\frac{dy}{dx} = 0$

4. Find $\frac{dy}{dx}$ for the curve $y = x^2 - 3x$. For what value of x is the gradient equal to 0?

   Solution: $\frac{dy}{dx} = 2x - 3$. Gradient $= 0$ when $\frac{dy}{dx} = 0$ and $x = \frac{3}{2}$

5. Use a table of derivatives to find $\frac{dz}{dt}$ when z is given by:

   (a) $z = 5t^3$
   Solution: $\frac{dz}{dt} = 15t^2$

   (b) $z = \sqrt{t}$
   Solution: $\frac{dz}{dt} = \frac{1}{2}t^{-\frac{1}{2}} = \frac{1}{2\sqrt{t}}$

   (c) $z = 3\sin(t)$
   Solution: $\frac{dz}{dt} = 3\cos t$

   (d) $z = 4e^{2t}$
   Solution: $\frac{dz}{dt} = 8e^{2t}$

6. Differentiate $y = 6\sin(2x) + 3x^2 - 5e^{3x}$

   Solution: $\frac{dy}{dx} = 12\cos 2x + 6x - 15e^{3x}$

7. If $\frac{dy}{dx} = 2x + 5x^4 + 3$, integrate the expression to find y.

   Solution: $y = x^2 + x^5 + 3x + c$

8. What is the constant of integration and why do you need it?

   Solution: The constant of integration is a constant which is always added to the expression when integrating. It is required because, when differentiating a constant disappears, so when integrating (reversing the process of differentiating) it is necessary to put in a constant.

9. Integrate with respect to x (a) $x^5 - 2x^3$ (b) $\frac{1}{x^4}$

   Solution: (a) $\frac{x^6}{6} - \frac{x^4}{2} + c$, (b) $-\frac{1}{3x^3} + c$

10. Find (a) $\int x^3\,dx$ (b) $\int \cos 2t\,dt$

    Solution:(a) $\frac{x^4}{4} + c$, (b) $\frac{1}{2}\sin 2t + c$