Loughborough University

Week 5: More Calculus, Integration

Try these exercises now, do not use a calculator, and try to solve the exercises without help

1. In each case, find any values of x for which $\frac{dy}{dx} = 0$

   $y = x^2 + 6x$
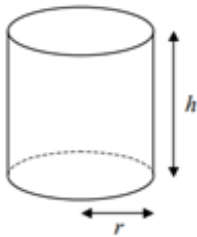
   $y = 4x^2 + 2x + 1$

   $y = x^3 - 12x$

   $y = 4 + 9x^2 - x^3$

2. Find the coordinates of any stationary points on each curve.

   $y = x^2 + 2x$

   $y = 5x^2 - 4x + 1$

3. The diagram shows a closed plastic cylinder used for making compost.

   The radius of the base and the height of the cylinder are r cm and h cm respectively and the surface area of the cylinder is 30 000 cm^2

   (a) Show that the volume of the cylinder, V cm^3 , is given by $V = 15000 - \pi r^3$.

   (b) Find the maximum volume of the cylinder and show that your value is a maximum.

4. Integrate with respect to y: $y^{\frac{1}{2}}$
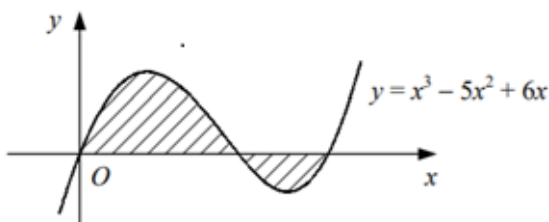
5. Find $\int y\, dx$ when

   (a) $y = 3x^2 - x + 6$

   (b) $y = x^6 - x^3 + 2x - 5$

   (c) $\sin 2x + 3\cos 3x$

   (d) $y = -e^{2x} + \frac{4}{x}$

6. The diagram shows the curve with the equation $y = x^3 - 5x^2 + 6x$.

   $y = x^3 - 5x^2 + 6x$

(a) Find the coordinates of the points where the curve crosses the x-axis.

(b) Show that the total area of the shaded regions enclosed by the curve and the x-axis is $3\frac{1}{12}$

7. Evaluate

(a) $\int_2^3 \frac{1}{x^2}dx$;

(b) $\int_0^{\frac{\pi}{3}} \cos 2x\,dx$;

(c) $\int_1^3 e^{2t}dt$.