Loughborough University

Week 4: Exponentials and Logarithms

Solutions

1. Simplify the expression $\frac{(5a^m)^2 a^2}{(a^3)^2}$

   Solution: $25a^{2m-4}$

2. Use a calculator to evaluate (a) e^1 (b) e^5 (c) e^{-5} (d) e^0

   Solution: To 4 s.f. (a) 2.718, (b) 148.4 (c) 0.006738 (d) 1

3. Calculate the values of the functions $\cosh(x) = \frac{e^x + e^{-x}}{2}$ and $\sinh(x) = \frac{e^x - e^{-x}}{2}$ for $x = 1, 0$ and -1

   Solution: $\cosh(1) = 1.543, \sinh(1) = 1.175, \cosh(0) = 1, \sinh(0) = 0, \cosh(-1) = 1.543, \sinh(-1) = -1.175$

4. Rewrite (a) $y = a^b$ in terms of logarithms, and (b) $\log_x(y) = p$ in exponential form

   Solution: (a) $\log_a y = b$, (b) $x^p = y$

5. Which of the following expressions are equivalent?

   $a = x^b \quad b = x^a \quad x = a^b \quad \log_x(a) = b \quad \log_a(x) = b \quad \log_x(b) = a$

   Solution:

   $a = x^b$ and $\log_x(a) = b$

   $b = x^a$ and $\log_x(b) = a$

   $x = a^b$ and $\log_a(x) = b$

6. Write $\ln(c) = d$ in exponential form.

   Solution: $e^d = c$

7. Simplify (without using a calculator) $\log_{10}(\frac{1}{10}) - \log_{10}(\frac{10}{27}) + \log_{10}(1000)$

   Solution: $1 + \log_{10} 27$

8. Simplify (without using a calculator) $2\ln(3) + \ln(4) - 2\ln(6)$

   Solution: $\ln(1) = 0$

9. Simplify $a^{\log_a x}$ and $e^{\ln x}$

   Solution: $a^{\log_a x} = x$ and $e^{\ln x} = x$

10. Solve for n by taking logs of both sides of the equation $1.04^n = 2$

    Solution: $n = 17.67$