# Loughborough University

# Week 3: Partial Fractions, Exponentials, Logarithms

## Solutions

Express in terms of partial fractions

1. $\frac{4x+5}{(x+3)(2x-1)}$

   **Solution:** $\frac{1}{x+3} + \frac{2}{2x-1}$

2. $\frac{s+2}{(s+1)^2}$

   **Solution:** $\frac{1}{s+1} + \frac{1}{(s+1)^2}$

3. $\frac{3x+1}{(x^2+x+10)(x-1)}$

   **Solution:** $\frac{-x+7}{3(x^2+x+10)} + \frac{1}{3(x-1)}$

4. $\frac{x^2+4x-3}{x^2+2x-3}$ What had to be done before this expression could be written in partial fractions?

   **Solution:** $1 + \frac{3}{2(x+3)} + \frac{1}{2(x-1)}$. Since the numerator is of the same order as the denominator (both $x^2$) we must write $\frac{x^2+4x-3}{x^2+2x-3}$ as $1 + \frac{2x}{x^2+2x-3}$ first. The denominator then has to be factorised.

5. Use the $e^x$ button on a calculator to find the values of the functions $\cosh(x) = \frac{e^x+e^{-x}}{2}$ and $\sinh(x) = \frac{e^x-e^{-x}}{2}$ for $x = 1, 0$ and $-1$

   **Solution:** $\cosh(1) = 1.543, \sinh(1) = 1.175, \cosh(0) = 1, \sinh(0) = 0, \cosh(-1) = 1.543, \sinh(-1) = -1.175$

6. Which of the following expressions are equivalent?

   $a = x^b \quad b = x^a \quad x = a^b \quad \log_x(a) = b \quad \log_a(x) = b \quad \log_x(b) = a$

   **Solution:**

   $a = x^b$ and $\log_x(a) = b$

   $b = x^a$ and $\log_x(b) = a$

   $x = a^b$ and $\log_a(x) = b$

7. Write $\ln(c) = d$ in exponential form.

   **Solution:** $e^d = c$

8. Simplify (without using a calculator) $\log_{10}(\frac{1}{10}) - \log_{10}(\frac{10}{27}) + \log_{10}(1000)$

   **Solution:** $1 + \log_{10} 27$

9. Simplify (without using a calculator) $2\ln(3) + \ln(4) - 2\ln(6)$

   **Solution:** $\ln(1) = 0$

10. Simplify $a^{\log_a x}$ and $e^{\ln x}$

    **Solution:** $a^{\log_a x} = x$ and $e^{\ln x} = x$

11. Solve for n by taking logs of both sides of the equation $1.04^n = 2$

    **Solution:** $n = 17.67$