Loughborough University

Week 1: Basic Algebra

Solutions

1. Write $2\frac{3}{8}$ as a decimal.

   Solution: 2.375.

2. What do each of the digits after the decimal point in the number 12.5467 represent?

   Solution: $\frac{5}{10}$, $\frac{4}{100}$, $\frac{6}{1000}$, $\frac{7}{10000}$, i.e. 5 tenths, 4 hundredths, 6 thousandths, 7 ten thousandths.

3. Do you expect your answer to $\frac{3}{8} \times \frac{4}{5}$ to be greater than $\frac{1}{2}$? Why or why not? Calculate the answer.

   Solution: Since $\frac{3}{8}$ is less than $\frac{1}{2}$ and $\frac{4}{5}$ is less than 1, then $\frac{3}{8} \times \frac{4}{5} < \frac{1}{2} \times 1 \leq \frac{1}{2}$. In fact $\frac{3}{8} \times \frac{4}{5} = \frac{12}{40} = \frac{3}{10}$.

4. Will the result of $\frac{7}{10} + \frac{3}{5}$ be greater than 1? Calculate the answer.

   Solution: Both fractions are greater than $\frac{1}{2}$, so adding them together will give an answer grater than 1. $\frac{7}{10} + \frac{3}{5} = \frac{7}{10} + \frac{6}{10} = \frac{13}{10} = 1\frac{3}{10}$.

5. Calculate $\frac{11}{20} - \frac{1}{3}$.

   Solution: $\frac{11}{20} - \frac{1}{3} = \frac{33}{60} - \frac{20}{60} = \frac{13}{60}$.

6. Simplify $5^7 \div 5^4$.

   Solution: Division by powers of the same number, so subtract the indices: $5^7 \div 5^4 = 5^3$.

7. Calculate a value for 4^{-2}.

   Solution: $4^{-2} = \frac{1}{4^2} = \frac{1}{16} = 0.0625$.

8. Calculate the value of $7^0 + 12^1$.

   Solution: $7^0 + 12^1 = 1 + 12 = 13$.

9. Evaluate $2x^3$ *when* $x = -1$

   Solution: when $x = -1$, $2x^3 = 2 \times (-1)^3 = 2 \times -1 = -2$

10. Simplify $2x^3 + x^3 + x^4 + x^2 + \frac{x^2}{x^6} - (x^2)^3$

    Solution: $2x^3 + x^3 + x^4 + x^2 \times x + \frac{x^2}{x^6} - (x^2)^3 = 3x^3 + x^4 + x^3 + x^{-4} - x^6 = 4x^3 + x^4 + x^{-4} - x^6$

11. Explain the distinction, if any, between each of the following expressions, and simplify if possible.

    (a) $4x - 2x$, (b) $4x(-2x)$, (c) $4x(2x)$, (d) $-4x(2x)$, (e) $-4x - 2x$, (f) $(4x)(2x)$

    Solution: (a) subtract to get $2x$, (b) multiply to get $-8x^2$, (c) multiply to get $8x^2$, (d) multiply to get $-8x^2$, (e) subtract to get $-6x$, (f) multiply to get $8x^2$

12. Explain the distinction between $(x + 3)(x + 2)$ and $x + 3(x + 2)$.

    Solution: In the first expression we are multiplying the result of $(x + 2)$ by the result of $(x + 3)$. In the second expression $(x + 2)$ is only multiplied by 3.

    $(x + 3)(x + 2) = x^2 + 5x + 6$ whereas $x + 3(x + 2) = 4x + 6$

13. Explain why x^2 is a factor of $4x^2 + 3yx^3 + 5yx^4$ but y is not, then factorise the $4x^2 + 3yx^3 + 5yx^4$

    Solution: Powers of x of at least order 2 appear in each term so x^2 is a factor. y does not appear in the first term so is NOT a factor. $4x^2 + 3yx^3 + 5yx^4 = x^2(4 + 3yx + 5yx^2)$

14. Factorise (a) $6x^2 + 7x - 5$ and (b) $4x^2 - 9$

    Solution: (a) $6x^2 + 7x - 5 = (2x - 1)(3x + 5)$ and (b) $4x^2 - 9 = (2x + 3)(2x - 3)$