Loughborough University

Week 1: Algebra Basics

Solutions

1. Do you expect your answer to $\frac{3}{8} \times \frac{4}{5}$ to be greater than $\frac{1}{2}$? Why or why not? Calculate the answer.

   Solution: Since $\frac{3}{8}$ is less than $\frac{1}{2}$ and $\frac{4}{5}$ is less than 1, then $\frac{3}{8} \times \frac{4}{5} < \frac{1}{2} \times 1 \leq \frac{1}{2}$. In fact $\frac{3}{8} \times \frac{4}{5} = \frac{12}{40} = \frac{3}{10}$.

2. Calculate the value of $7^0 + 12^1$.

   Solution: $7^0 + 12^1 = 1 + 12 = 13$.

3. Explain the distinction, if any, between each of the following expressions, and simplify if possible.

   (a) $4x - 2x$, (b) $4x(-2x)$, (c) $4x(2x)$, (d) $-4x(2x)$, (e) $-4x - 2x$, (f) $(4x)(2x)$

   Solution: (a) subtract to get $2x$, (b) multiply to get $-8x^2$, (c) multiply to get $8x^2$, (d) multiply to get $-8x^2$, (e) subtract to get $-6x$, (f) multiply to get $8x^2$

4. Explain the distinction between $(x + 3)(x + 2)$ and $x + 3(x + 2)$.

   Solution: In the first expression we are multiplying the result of $(x + 2)$ by the result of $(x + 3)$. In the second expression $(x + 2)$ is only multiplied by 3.

5. Factorise (a) $6x^2 + 7x - 5$ and (b) $4x^2 - 9$

   Solution: $6x^2 + 7x - 5 = (2x - 1)(3x + 5)$ and $4x^2 - 9 = (2x + 3)(2x - 3)$

6. Simplify, if possible, $\frac{x^2+2x-15}{2x^2-5x-3}$

   Solution: $\frac{x^2+2x-15}{2x^2-5x-3} = \frac{(x+5)(x-3)}{(2x+1)(x-3)} = \frac{x+5}{2x+1}$

7. Transpose $v = \sqrt{x + 2y}$, (a) for x, (b) for y.

   Solution: (a) $x = v^2 - 2y$ (b) $y = (v^2 - x)/2$

8. The surface area of a sphere is given by the formula $SA = 4\pi r^2$. If the sphere has a surface area of 20 cm^2, what is the radius of the sphere?

   Solution: $r = 1.26$ cm