

# Engineering Maths

## First Aid Kit

2.2

## Negative and fractional powers

### Introduction

Sometimes it is useful to use negative and fractional powers. These are explained on this leaflet.

### 1. Negative powers

Sometimes you will meet a number raised to a negative power. This is interpreted as follows:

$$a^{-m} = \frac{1}{a^m}$$

This can be rearranged into the alternative form:

$$a^m = \frac{1}{a^{-m}}$$

### Example

$$3^{-2} = \frac{1}{3^2}, \quad \frac{1}{5^{-2}} = 5^2, \quad x^{-1} = \frac{1}{x^1} = \frac{1}{x}, \quad x^{-2} = \frac{1}{x^2}, \quad 2^{-5} = \frac{1}{2^5} = \frac{1}{32}$$

### Exercises

1. Write the following using only positive powers:

a)  $\frac{1}{x^{-6}}$ ,   b)  $x^{-12}$ ,   c)  $t^{-3}$ ,   d)  $\frac{1}{4^{-3}}$ ,   e)  $5^{-17}$ .

2. Without using a calculator evaluate a)  $2^{-3}$ ,   b)  $3^{-2}$ ,   c)  $\frac{1}{4^{-2}}$ ,   d)  $\frac{1}{2^{-3}}$ ,   e)  $\frac{1}{4^{-3}}$ .

### Answers

1. a)  $x^6$ ,   b)  $\frac{1}{x^{12}}$ ,   c)  $\frac{1}{t^3}$ ,   d)  $4^3$ ,   e)  $\frac{1}{5^{17}}$ .

2. a)  $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ ,   b)  $\frac{1}{9}$ ,   c) 16,   d) 32,   e) 64.

### 2. Fractional powers

To understand fractional powers you first need to have an understanding of roots, and in particular square roots and cube roots. If necessary you should consult leaflet 1.2 *Powers and Roots*.

When a number is raised to a fractional power this is interpreted as follows:

$$a^{1/n} = \sqrt[n]{a}$$

So,

$a^{1/2}$  is a square root of  $a$

$a^{1/3}$  is the cube root of  $a$

$a^{1/4}$  is a fourth root of  $a$

### Example

$$3^{1/2} = \sqrt{3}, \quad 27^{1/3} = \sqrt[3]{27} \text{ or } 3, \quad 32^{1/5} = \sqrt[5]{32} = 2, \\ 64^{1/3} = \sqrt[3]{64} = 4, \quad 81^{1/4} = \sqrt[4]{81} = 3$$

Fractional powers are useful when we need to calculate roots using a scientific calculator. For example to find  $\sqrt[7]{38}$  we rewrite this as  $38^{1/7}$  which can be evaluated using a scientific calculator. You may need to check your calculator manual to find the precise way of doing this, probably with the buttons  $x^y$  or  $x^{1/y}$ .

Check that you are using your calculator correctly by confirming that

$$38^{1/7} = 1.6814 \quad (4 \text{ dp})$$

More generally  $a^{m/n}$  means  $\sqrt[n]{a^m}$ , or equivalently  $(\sqrt[n]{a})^m$ .

$$a^{m/n} = \sqrt[n]{a^m} \text{ or equivalently } (\sqrt[n]{a})^m$$

### Example

$$8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4, \quad \text{and} \quad 32^{3/5} = (\sqrt[5]{32})^3 = 2^3 = 8$$

### Exercises

1. Use a calculator to find a)  $\sqrt[5]{96}$ , b)  $\sqrt[4]{32}$ .
2. Without using a calculator, evaluate a)  $4^{3/2}$ , b)  $27^{2/3}$ .
3. Use the third law of indices to show that

$$a^{m/n} = \sqrt[n]{a^m}$$

and equivalently

$$a^{m/n} = (\sqrt[n]{a})^m$$

### Answers

1. a) 2.4915, b) 2.3784. 2. a)  $4^{3/2} = 8$ , b)  $27^{2/3} = 9$ .