

## Rearranging formulas 2

### Introduction

This leaflet develops the work started on leaflet 2.11, and shows how more complicated formulas can be rearranged.

### 1. Further transposition

Remember that when you are trying to rearrange, or **transpose**, a formula, the following operations are allowed.

- add or subtract the same quantity to or from both sides
- multiply or divide both sides by the same quantity

A further group of operations is also permissible.

A formula remains balanced if we perform the same operation to both sides of it. For example, we can square both sides, we can square-root both sides. We can find the logarithm of both sides. Study the following examples.

#### Example

Transpose the formula  $p = \sqrt{q}$  to make  $q$  the subject.

#### Solution

Here we need to obtain  $q$  on its own. To do this we must find a way of removing the square root sign. This can be achieved by squaring both sides since

$$(\sqrt{q})^2 = q$$

So,

$$\begin{aligned} p &= \sqrt{q} \\ p^2 &= q \quad \text{by squaring both sides} \end{aligned}$$

Finally,  $q = p^2$ , and we have succeeded in making  $q$  the subject of the formula.

#### Example

Transpose  $p = \sqrt{a+b}$  to make  $b$  the subject.

### Solution

$$\begin{aligned}p &= \sqrt{a+b} \\p^2 &= a+b && \text{by squaring both sides} \\p^2 - a &= b\end{aligned}$$

Finally,  $b = p^2 - a$ , and we have succeeded in making  $b$  the subject of the formula.

### Example

Make  $x$  the subject of the formula  $v = \frac{k}{\sqrt{x}}$ .

### Solution

$$\begin{aligned}v &= \frac{k}{\sqrt{x}} \\v^2 &= \frac{k^2}{x} && \text{by squaring both sides} \\xv^2 &= k^2 && \text{by multiplying both sides by } x \\x &= \frac{k^2}{v^2} && \text{by dividing both sides by } v^2\end{aligned}$$

and we have succeeded in making  $x$  the subject of the formula.

### Example

Transpose the formula  $T = 2\pi\sqrt{\frac{\ell}{g}}$  for  $\ell$ .

### Solution

This must be carried out carefully, in stages, until we obtain  $\ell$  on its own.

$$\begin{aligned}T &= 2\pi\sqrt{\frac{\ell}{g}} \\ \frac{T}{2\pi} &= \sqrt{\frac{\ell}{g}} && \text{by dividing both sides by } 2\pi \\ \left(\frac{T}{2\pi}\right)^2 &= \frac{\ell}{g} && \text{by squaring both sides} \\ \ell &= g\left(\frac{T}{2\pi}\right)^2\end{aligned}$$

### Exercises

1. Make  $r$  the subject of the formula  $V = \frac{4}{3}\pi r^3$ .
2. Make  $x$  the subject of the formula  $y = 4 - x^2$ .
3. Make  $s$  the subject of the formula  $v^2 = u^2 + 2as$

### Answers

1.  $r = \sqrt[3]{\frac{3V}{4\pi}}$ .
2.  $x = \sqrt{4 - y}$ .
3.  $s = \frac{v^2 - u^2}{2a}$ .