Rearranging formulas 2

Introduction

This leaflet develops the work started on leaflet 2.13, and shows how more complicated formulas can be rearranged.

Further transposition

Remember that when you are trying to rearrange, or **transpose**, a formula, the following operations are allowed.

- add or subtract the same quantity to or from both sides
- multiply or divide both sides by the same quantity

A further group of operations is also permissible.

A formula remains balanced if we perform the same operation to both sides of it. For example, we can square both sides, we can square-root both sides. We can find the logarithm of both sides. Study the following examples.

Example
Transpose the formula $p = \sqrt{q}$ to make q the subject.

Solution
Here we need to obtain q on its own. To do this we must find a way of removing the square root sign. This can be achieved by squaring both sides since

$$(\sqrt{q})^2 = q$$

So,

$$\begin{aligned} p &= \sqrt{q} \\ p^2 &= q \qquad \text{by squaring both sides} \end{aligned}$$

Finally, $q = p^2$, and we have succeeded in making q the subject of the formula.

Example
Transpose $p = \sqrt{a+b}$ to make b the subject.

Solution

$$\begin{aligned} p &= \sqrt{a+b} \\ p^2 &= a+b \qquad \text{by squaring both sides} \\ p^2 - a &= b \end{aligned}$$

Finally, $b = p^2 - a$, and we have succeeded in making b the subject of the formula.

Example
Make x the subject of the formula $v = \frac{k}{\sqrt{x}}$.

Solution

$$\begin{aligned} v &= \frac{k}{\sqrt{x}} \\ v^2 &= \frac{k^2}{x} \qquad \text{by squaring both sides} \\ xv^2 &= k^2 \qquad \text{by multiplying both sides by } x \\ x &= \frac{k^2}{v^2} \qquad \text{by dividing both sides by } v^2 \end{aligned}$$

and we have succeeded in making x the subject of the formula.

Example
Transpose the formula $R = Q(1+i)^3$ for i.

Solution
This must be carried out carefully, in stages, until we obtain i on its own.

$$\begin{aligned} R &= Q(1+i)^3 \\ \frac{R}{Q} &= (1+i)^3 \qquad \text{by dividing both sides by } Q \\ \sqrt[3]{\frac{R}{Q}} &= 1+i \qquad \text{by taking the cube root of both sides} \\ i &= \sqrt[3]{\frac{R}{Q}} - 1 \qquad \text{by subtracting 1 from each side} \end{aligned}$$

Exercises

1. Make r the subject of the formula $V = \frac{4}{3}\pi r^3$.
2. Make x the subject of the formula $y = 4 - x^2$.
3. Make s the subject of the formula $v^2 = u^2 + 2as$
4. Make P the subject of the formula $S = P(1+i)^n$. Try making i the subject.

Answers

1. $r = \sqrt[3]{\frac{3V}{4\pi}}$. 2. $x = \sqrt{4-y}$. 3. $s = \frac{v^2-u^2}{2a}$. 4. $P = \frac{S}{(1+i)^n}$. $i = \sqrt[n]{\frac{S}{P}} - 1$

