Factorials !

Introduction

In many engineering calculations you will come across the symbol ! which you may not have met before in mathematics classes. This is known as a **factorial**. The factorial is a symbol which is used when we wish to multiply consecutive whole numbers together, as you will see below.

1. Factorials

The number $5 \times 4 \times 3 \times 2 \times 1$ is written as 5!, which is read as 'five factorial'. If you actually perform the multiplication you will find that $5! = 120$. Similarly $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$ which equals 5040. A rather special case is 0!. This is defined to equal 1 and this might seem somewhat strange. Just learn this!

You will not be required to find factorials of negative numbers or fractions.

Factorials are used in **combination notation** which arises frequently in probability theory. The notation $\binom{n}{r}$ stands for $\frac{n!}{(n-r)!r!}$. For example

$$\binom{6}{4} = \frac{6!}{(6-4)!4!} = \frac{6!}{2!4!}$$

Exercises

1. Without using a calculator evaluate 2!, 3! and 4!.

2. Show that $\frac{5!}{3!}$ equals 20.

3. Explain why $n! = n \times (n-1)!$ for any positive whole number n.

4. Explain why $\dfrac{n!}{(n-1)!} = n$ for any positive whole number n.

5. Evaluate a) $\binom{9}{3}$, b) $\binom{5}{2}$, c) $\binom{6}{1}$.

Answers
1. $2! = 2$. $3! = 6$ and $4! = 24$. Note that $3! = 3 \times 2!$, and that $4! = 4 \times 3!$.

5. a) 84, b) 10, c) 6.

2. Using a calculator to find factorials

Your scientific calculator will be pre-programmed to find factorials. Look for a button marked !, or consult your calculator manual. Check that you can use your calculator to find factorials by verifying that

$$10! = 3628800$$