Fractions

Introduction

The ability to work confidently with fractions, both number fractions and algebraic fractions, is an essential skill which underpins all other algebraic processes. In this leaflet we remind you of how number fractions are simplified, added, subtracted, multiplied and divided.

Expressing a fraction in its simplest form

In any **fraction**, $\dfrac{p}{q}$ say, the number p at the top is called the **numerator**. The number q at the bottom is called the **denominator**. The number q must never be zero. A fraction can always be expressed in different, yet **equivalent** forms. For example, the two fractions $\frac{2}{6}$ and $\frac{1}{3}$ are equivalent. They represent the same value. A fraction is expressed in its **simplest form** by cancelling any <u>factors</u> which are common to both the numerator and the denominator. You need to remember that factors are numbers which are <u>multiplied</u> together. We note that

$$\frac{2}{6} = \frac{1 \times 2}{2 \times 3}$$

and so there is a factor of 2 which is common to both the numerator and the denominator. This common factor can be cancelled to leave the equivalent fraction $\dfrac{1}{3}$. Cancelling is equivalent to dividing the top and the bottom by the common factor.

Example

$\dfrac{12}{20}$ is equivalent to $\dfrac{3}{5}$ since $\dfrac{12}{20} = \dfrac{4 \times 3}{4 \times 5} = \dfrac{3}{5}$

Exercises

1. Express each of the following fractions in its simplest form:

a) $\frac{12}{16}$,   b) $\frac{14}{21}$,   c) $\frac{3}{6}$,   d) $\frac{100}{45}$,   e) $\frac{7}{9}$,   f) $\frac{15}{55}$,   g) $\frac{3}{24}$.

Answers

1. a) $\frac{3}{4}$,   b) $\frac{2}{3}$,   c) $\frac{1}{2}$,   d) $\frac{20}{9}$,   e) $\frac{7}{9}$,   f) $\frac{3}{11}$,   g) $\frac{1}{8}$.

Addition and subtraction of fractions

To add two fractions we first re-write each fraction so that they both have the same denominator. This denominator is chosen to be the **lowest common denominator**. This is the smallest number which is a multiple of both denominators. Then, the numerators only are added, and the result is divided by the lowest common denominator.

Example

Simplify a) $\frac{7}{16} + \frac{5}{16}$,   b) $\frac{7}{16} + \frac{3}{8}$.

Solution

a) In this case the denominators of each fraction are already the same. The lowest common denominator is 16. We perform the addition by simply adding the numerators and dividing the result by the lowest common denominator. So, $\frac{7}{16} + \frac{5}{16} = \frac{7+5}{16} = \frac{12}{16}$. This answer can be expressed in the simpler form $\frac{3}{4}$ by cancelling the common factor 4.

b) To add these fractions we must rewrite them so that they have the same denominator. The lowest common denominator is 16 because this is the smallest number which is a multiple of both denominators. Note that $\frac{3}{8}$ is equivalent to $\frac{6}{16}$ and so we write $\frac{7}{16} + \frac{3}{8} = \frac{7}{16} + \frac{6}{16} = \frac{13}{16}$.

Example
Find $\frac{1}{2} + \frac{2}{3} + \frac{4}{5}$.

Solution
The smallest number which is a multiple of the given denominators is 30. We express each fraction with a denominator of 30.
$$\frac{1}{2} + \frac{2}{3} + \frac{4}{5} = \frac{15}{30} + \frac{20}{30} + \frac{24}{30} = \frac{59}{30}$$

Exercises
1. Evaluate each of the following:

a) $\frac{2}{3} + \frac{5}{4}$,   b) $\frac{4}{9} - \frac{1}{2}$,   c) $\frac{3}{4} + \frac{5}{6}$,   d) $\frac{1}{4} + \frac{1}{3} + \frac{1}{2}$,   e) $\frac{2}{5} - \frac{1}{3} - \frac{1}{10}$,   f) $\frac{4}{5} + \frac{1}{3} - \frac{3}{4}$.

Answers
1. a) $\frac{23}{12}$,   b) $-\frac{1}{18}$,   c) $\frac{19}{12}$,   d) $\frac{13}{12}$,   e) $-\frac{1}{30}$,   f) $\frac{23}{60}$.

Multiplication and division of fractions

Multiplication of fractions is more straightforward. We simply multiply the numerators to give a new numerator, and multiply the denominators to give a new denominator. For example
$$\frac{5}{7} \times \frac{3}{4} = \frac{5 \times 3}{7 \times 4} = \frac{15}{28}$$

Division is performed by inverting the second fraction and then multiplying. So,
$$\frac{5}{7} \div \frac{3}{4} = \frac{5}{7} \times \frac{4}{3} = \frac{20}{21}$$

Exercises
1. Find a) $\frac{4}{26} \times \frac{13}{7}$,   b) $\frac{2}{11} \div \frac{3}{5}$,   c) $\frac{2}{1} \times \frac{1}{2}$,   d) $\frac{3}{7} \times \frac{2}{5}$,   e) $\frac{3}{11} \times \frac{22}{5}$,   f) $\frac{5}{6} \div \frac{4}{3}$.

Answers
1. a) $\frac{2}{7}$,   b) $\frac{10}{33}$,   c) 1,   d) $\frac{6}{35}$,   e) $\frac{6}{5}$,   f) $\frac{5}{8}$.