MEC Annual Symposium – 24th May, 2017

Algebra: Gateway to mathematics, science, society and the world

Keynote Speakers

Prof. Luis Radford - Laurentian University, Canada

On algebraic thinking

Abstract

In this presentation I draw on the history of mathematics, semiotics, and mathematics education research to deal with the problem of elementary algebraic thinking. I suggest that two distinctive interrelated features of algebraic thinking are: (1) its peculiar way to deal with unknown quantities, and (2) the specific culturally and historically evolved modes of representing/symbolizing the unknown quantities and their operations. While the first feature refers to the *analytic* manner in which calculations are carried out with known and unknown quantities, the second feature refers to the constraints and affordances of the diverse *semiotic systems* (e.g., pre-alphanumeric, alphanumeric, graphic) through which unknown quantities are conceptualized and represented. These two distinctive features of algebraic thinking allow us to better understand the students' legendary difficulties in the learning of algebra and provide us with clues to tackle the specifics of task design. Some videotaped classroom excerpts will serve to illustrate the main ideas of the presentation.

Prof. Paul Drijvers - Utrecht University, Netherlands

Algebra artefacts

Abstract

Digital tools for algebra are widely available. On the one hand, the opportunity to outsource most algebraic work to such tools questions the legitimacy of current algebra curricula. On the other hand, the question is of how to "exploit" such tools for educational purpose: can students learn algebra efficiently through the use of digital technology? In this presentation, these questions will be addressed through examples, and findings from qualitative and quantitative studies.

Prof. Jeremy Hodgen – Nottingham University, UK

Algebraic understanding

In this talk, I will discuss how lower secondary students' understanding of, and progression in, algebra. I will draw on evidence from a nationally representative sample of students in England conducted as part of the Increasing Competence and Confidence in Algebra and Multiplicative Structures (ICCAMS) study. I will compare this to the Concepts in Secondary Mathematics and Science study on which the study was based. In addition, I will discuss the design of tasks aimed at developing algebraic understanding taken from a current large-scale intervention study.

Dr Dave Hewitt, Loughborough University, UK

Learner difficulties with algebra: are they really about algebra and do they really have to be difficult?

Abstract

There have been several studies from the 1980s onwards which have identified difficulties that many learners have with algebra (e.g. Küchemann, 1981; Herscovics, 1989; Cooper et al., 1997). Yet some studies in the 2000s give a clear sense that relatively young learners can successfully work with algebraic situations (e.g. Carraher et al., 2001; Schliemann et al., 2003). So what is difficult about working algebraically? Are there other factors which affect success with algebraic tasks but which are not essentially algebraic in nature? In this talk I will present an argument that not all difficulties with algebra are to do with algebra, and that there may be more productive ways to go about teaching algebra than are commonly found in many school classrooms.

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Seminar Speakers – 25th May

Session 1

Prof. Despina Potari – University of Athens, Greece

Mathematics Teacher Collaboration in Greece: Examples from algebra teaching

Teacher Collaboration is not common among the Greek mathematics teachers. However, during the last years different collaborative groups have been established through initiatives taken by the teachers and often supported from school advisors and the universities. In the presentation, I will offer some examples from such collaborations where the main focus is the teaching and learning of algebra at secondary school. In particular, I will offer examples of the activities of a collaborative group called "The laboratory of algebra teaching" and address issues related to the impact that these had on teachers' professional growth.

Prof. Kirsti Hemmi[1,2] and Yvonne Liljekvist[2,3] [1]Åbo Akademi, [2] Uppsala University, [3]Karlstad University

Resources for algebra teaching and the teachers' openness for change: The Swedish case

In Sweden, algebra has been included in the mathematics curriculum from the first school grades since several decades and early algebra has been in focus of teacher education as well. Still Swedish students' results in algebra in national and international evaluations have remained low. In search for the reasons for the failure we investigate traditions developed in different educational arenas; the formulation arenas producing steering documents and curriculum materials, and the realization arenas where teachers develop and maintain their own more or less tacit traditions. This study focuses on the latter. We investigate how teachers in compulsory school talk about algebra teaching and the expected progress of students. The data consists of eight focus group interviews (2 hours) from different schools (38 teachers, mean 16 years of practice, SD=9,4). The interview guide contained open questions related to algebra learning and teaching, as well as an intervention with mathematical tasks connected to the "big ideas" of algebra identified by earlier research as important to develop from early age. In the seminar, we will present our initial ideas and themes from the first data analyses.

Dr Douglas Butler – Director, TSM workshops and TSM Resources, and co-author of Autograph debutler@argonet.co.uk

Using dynamic software to enhance understanding of algebraic concepts

Using letters and changing their value can have a dramatic effect on the visualization of a wide variety of topics at all levels of mathematics education. This paper will include:

- the link between completing the square and the turning point of a quadratic function
- the transformation of a normal distribution
- the locus of a calculated area under curve compared to the integral function
- the interpretation of a matrix transformation using parameters
- the algebra of vectors
- the algebra of complex numbers in 2D (the Argand Diagram) and 3D

The main premise is to achieve understanding through the correct prediction of the outcome when a parameter is changed. This is an important strategy whether the teacher is using one large projected image in front of the class or the students are controlling their own technology.

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Session 2

Dr Heidi Strømskag – Norwegian University of Science and Technology

Combining the theory of didactical situations and semiotic theory to investigate students' enterprise of representing a relationship in algebraic notation

This paper investigates conditions that enable or hinder students' opportunity to represent a relationship between percentage growth of length and area when looking at the enlargement of a square. The empirical material consists of a mathematical task and a video-recorded small-group session where three student teachers are solving the task (with teacher interaction). The observed session is taken from a compulsory mathematics course in an ordinary classroom (i.e., it is not the result of didactical engineering). It is chosen because: 1) it provides an example of an evolution of the milieu which enabled the students to develop the knowledge aimed at; and, 2) it shows the complexity and importance of changing representation register. The theory of didactical situations and semiotic theory are used as conceptual frameworks to investigate the possibilities and constraints that influence students' generational activity. It is shown how an evolution of the milieu enables the students to create manipulatives (plane geometrical figures) that are instrumental in the generalisation process. The analysis shows how use of different notation systems constrains the formulation phase, and further, how transformation of percentage and fractional notation into geometrical figures (belonging to a different semiotic register) enables the target mathematical knowledge to be expressed algebraically.

I will discuss how combination of the two theories is helpful in determining *didactical variables*—i.e., what semiotic systems should be used (in a particular case) and what values "fit" the chosen systems.

Jorunn Reinhardtsen – University of Agder, Norway

The evolution of classroom mathematical practices in the context of introductory algebra

This paper explores contingencies between teaching and learning in an introductory algebra classroom. We follow 6th grade students and their teacher in an American classroom through five consecutive lessons as they move beyond recursive descriptions of patterns to a structural approach. Patterns are often discussed in the literature as a way of introducing young students to early algebra, without involving the algebraic syntax. However, here we discuss patterns in the context of moving from arithmetic to algebra involving changes in both form and function of the mathematical discourse. This paper reports on classroom data from an international project called VIDEOMAT (see Kilhamn & Röj-Lindberg, 2013), involving the four countries Finland, Norway, Sweden and USA (California). The analyses done here are part of a larger study that investigate learning in algebra seen as consisting of three intertwined genetic processes, microgenesis, sociogenesis and ontogenesis, as described in Saxe (2012). In a previous study these data have been analyzed with a focus towards ontogenetic and microgenetic processes of learning in the introductory algebra classroom (Reinhardtsen & Givin, in press). The analyses done in this paper are informed by these findings as well as other research regarding patterning activities, and, more generally, the transition from arithmetic to algebra. The aim of this study is to identify characteristics regarding a sociogenetic process of learning introductory algebra through patterning activities. To describe the learning process at classroom level, the notion of *classroom mathematical practices* as developed by Cobb, Stephan, McClain and Gravemeijer (2001) is drawn upon. Further, learning is investigated as change in discourse employing Sfard's (2008) communicational approach. Preliminary findings indicate that the introduction of geometrical patterns in the classroom, after having worked with numerical patterns in which a recursive approach is prominent, initiate a shift in the discourse of both the teacher and students in which the increased use of ordinal numbers seem to spur a structural approach to patterns.

References

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Kilhamn, C., & Röj-Lindberg, A. S. (2013). Seeking hidden dimensions of algebra teaching through video analysis. In B. Grevholm, P. S. Hundeland, K. Juter, K. Kislenko, & P.-E. Persson (Eds.) *Nordic research in didactics of mathematic: Past, present and future* (pp. 299-328). Oslo: Cappelen Damm.

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Prof. Dr Reinhard Oldenburg – Augsburg, Germany, reinhard.oldenburg@math.uni-augsburg.de

Is there an Algebra Problem after all?

Many studies have highlighted the diverse problems students encounter in learning symbolic algebra. It seems obvious that between arithmetic and algebra there is a cut that presents specific obstacles for students and many examples of students struggling with the symbolic sign system have been documented. However, in a recent research project we tried to find students that perform well in numerical calculations but show problems when algebra gets symbolic. The hope was that the study of such students that show pure algebraic problems would be suitable to test interventions that try to cure the "algebra problem". However, none of the collaborating teachers of my university could spot such a student. This leads to a speculative hypothesis: *There is no cognitive demand specific to symbolic algebra*. This hypothesis is very speculative and there are pro and contra arguments that will be discussed.

Pro-arguments: Numbers and fractions present almost the same generality as variables, e.g. ½ is ½ of *some* unit. The coordination of signs (operators, parentheses) is essential as well in evaluating numerical expressions and the structural complexity is not different from symbolic algebra.

Contra arguments: Solving arithmetical and algebraic problems activates different areas of the brain. Numerical and symbolic items correlate only weakly in various tests.

These and other arguments are discussed and put together. Finally, it is suggested that an adequate answer depends of the art of algebra considered.