

Self-Explanation Training: A Guide for Mathematics Lecturers

Self-explanation training has been shown to significantly improve undergraduate students' comprehension of mathematical proofs. This document explains the need for such training and the evidence to date on its efficacy.

The Challenges of Proof Comprehension

Undergraduate students have to learn a lot from written proofs. Although lecturers explain these proofs, pointing out key logical inferences and overall proof structures, students are unlikely to grasp every detail during lecture time. As a result, much understanding must be developed through independent reading. So it is important that this reading is effective.

Unfortunately, new undergraduates have little experience with extended mathematical proofs, and research in mathematics education shows that many do not read well. Interview studies show that students are inclined to skip explanatory text, and to move through a mathematical argument without serious attempts to understand each inference or to sort out any confusion (Shepherd, Selden, & Selden, 2012; Weber, 2010). As a result, they are often unable to judge the validity of even simple purported proofs (Selden & Selden, 2003). A recent study has shed more light on such phenomena by using eye tracking, which allows real-time recording of an individual's focus of attention as they read text on a screen (Inglis & Alcock, 2012). This study compared the reading of first year undergraduates with that of professional mathematicians, and it confirmed that students pay proportionally more attention to the formulae in a proof, and less to the surrounding text. It also found that students move their attention around to a lesser extent: compared with mathematicians, they are less likely to make eye movements consistent with identifying why a given claim follows in a logically valid way from earlier statements.

Fortunately, self-explanation training has been shown to significantly improve the quality of students' reading and their resultant proof comprehension.

Self-Explanation Training for Mathematical Proofs

Self-explanation training has been shown to improve comprehension in a variety of subject areas (Ainsworth & Burcham, 2007; Chi, de Leeuw, Chiu, & Lavancher, 1994); the version that accompanies this guide is designed specifically to support learning in undergraduate mathematics. It comprises a short, simple self-study booklet that teaches students to question their understanding of each statement in a proof, to relate statements to each other and to their existing knowledge, and to distinguish self-explanations from less effective monitoring and paraphrasing comments. The booklet provides example self-explanations and practice proofs, and a typical student can expect to work through it in around 20 minutes.

Self-Explanation Training: The Evidence

A series of research studies conducted in the Mathematics Education Centre at Loughborough University has established that self-explanation training significantly improves undergraduate mathematics students' proof comprehension (Hodds, Alcock, & Inglis, 2014). In particular:

- Audio recording shows that who have studied the training produce significantly more high-quality explanations when reading a proof: they infer more justifications and give more explanations that show awareness of the proof's structure.
- Eye tracking shows that students who have studied the training concentrate harder when reading: their fixation durations (a measure of the intensity of concentration) are longer.
- Eye tracking shows that students who have studied the training are significantly more attentive to logical relationships between statements in a proof: they move their attention around more during their reading attempts.
- Proof comprehension tests (cf. Mejía-Ramos, Fuller, Weber, Rhoads, & Samkoff, 2012) show that students who have studied the training attain significantly better comprehension of a subsequent proof. In a lab-based study, scores of those who had studied the training were higher by almost one standard deviation.
- Proof comprehension tests administered with a delay show that the self-explanation effect is persistent: in a lecture-based study, students who has received the training scored significantly higher on an immediate comprehension test, *and* on a comprehension test for a different proof distributed almost three weeks later.

We are now planning further studies to investigate the effects of self-explanation training upon reading extended materials such as lecture notes, the interactions between self-explanation training and factors such as pre-existing subject knowledge and logical reasoning ability, and the effects of self-explanation training in collaborative learning situations. An updated version of this guide will be made available once these studies are completed.