

Loughborough University
Department of Mathematical Sciences
MATHEMATICAL CHALLENGE
CHRISTMAS - 2024

Problem 1. The 2-digit number 76 has the property that its square 5776 ends with the same 2-digit number. Find all the n -digit numbers with the same property for all $n \leq 10$.

How many such numbers exist for general n ? Propose an efficient way for their computation for arbitrary given n .

Problem 2. Define the integer sequences a_n, b_n, c_n, d_n from the relation

$$(1 + \sqrt{2} + \sqrt{3})^n = a_n + b_n\sqrt{2} + c_n\sqrt{3} + d_n\sqrt{6}.$$

Find the limits of the ratios $\frac{b_n}{a_n}, \frac{c_n}{b_n}, \frac{d_n}{c_n}$ as n goes to infinity.

Problem 3. The University of Lapland organised a Christmas football competition between its Departments, with each team playing a game against every other team. The rules are as usual: 3 points for a victory, 1 point for a draw and 0 points otherwise.

After the end of the tournament one of the teams was disqualified with all the points gained in games against that team to be removed from the record. One of the Departments complained that as a result of this, instead of being a clear winner, its team finished at the clear bottom of the table.

Could it be possible? How does the answer depend on the number of the Departments? Justify your answer.

Remarks.

1. There will be a first prize of £50 to the person handing in what will be considered to be the best effort to these problems. There may also be special prizes for the most original solutions.
2. Any student registered on one of the undergraduate programmes in the Department of Mathematical Sciences may submit solutions to any or all of these problems.
3. Solutions should be scanned and e-mailed on or before January 31, 2025 to either Prof. A.P. Veselov (A.P.Veselov@lboro.ac.uk) or Dr. B. Winn (B.Winn@lboro.ac.uk), who will be the judges for the Challenge.