Geometry of vortex equilibria and complex roots of Wronskians

PhD project

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The study of vortex dynamics in the plane is a very classical subject going back to Helmholtz. It is closely related to the theory of differential equations in the complex domain, see a nice review in [1].

In particular, one can construct special equilibrium configurations of vortices as the complex roots of the Wronskians $W(H_{i_1}, \dots, H_{i_n})$ of the classical Hermite polynomials:

$$H_0(x) = 1,\ H_1(x) = 2x,\ H_2(x) = 4x^2 - 2,\ H_3(x) = 8x^3 - 12x,\ H_4(x) = 16x^4 - 48x^2 + 12, \dots.$$

Such Wronskians appeared recently in many areas, including the theory of random matrices.

In some cases the geometry of such root configurations can be described using the Ferrers representation of the Young diagrams [2], but in general this question is widely open.

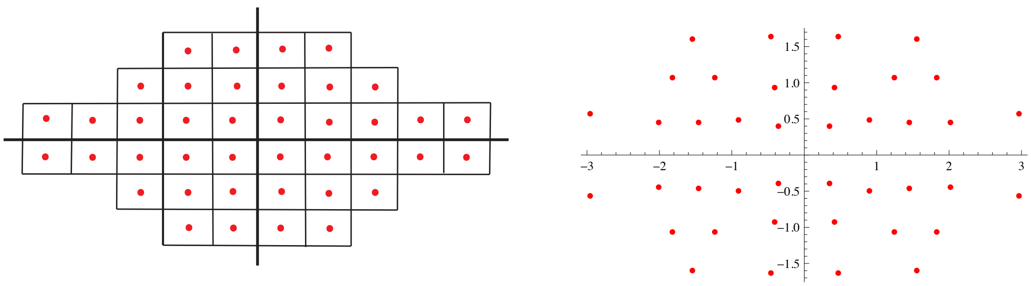

Figure 1: Ferrers diagram and the zeroes of the corresponding Wronskian from [2]

Similar situation is in the periodic setting describing the so-called vortex streets, which include the famous von Karman street [3].

The aim of the project to make some progress in this challenging direction using the tools from the theory of integrable systems, complex analysis and representation theory in combination with Maple/Mathematica experiments.

References

[1] H. Aref *Point vortex dynamics: A classical mathematics playground.* J. Math. Phys. **48** (2007), 065401.

[2] G. Felder, A.D. Hemery, A.P. Veselov *Zeroes of Wronskians of Hermite polynomials and Young diagrams.* Physica D **241** (2012), 2131-2137.

[3] A.D. Hemery, A.P. Veselov *Periodic vortex streets and complex monodromy.* SIGMA **10** (2014), 114.