Integrability of semi-discrete systems

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The subject of Integrable Systems started with the remarkable discovery of solitary waves - solitons - in a variety of physical models. Now Integrable systems are of great interest in modern mathematics, where they have important connections with algebra and differential geometry, and in physics, where they describe many important physical models. The theory of integrable systems gave rise to many important concepts in mathematical physics. These systems have remarkable properties such as infinite number of symmetries and conservation laws. Integrable systems quite often have fundamental and universal applications in mathematics and physics.

One of the most important and challenging problems in the theory of integrable systems is how to recognize when a given system is integrable. There are several approaches to this problem such as Painlevé theory, perturbative analysis of almost integrable and quasi-linear systems, multi-scale expansion theory, the symmetry approach. The latter is the closest to this project, and the focus of the project will be on integrability and integrability tests for **semi-discrete (**DΔEs**)** equations, i.e. equations whose independent variables are both continuous and discrete. The only complete classification result for Integrable DΔEs was obtained by Yamilov in 1983 for equations depending on three points on the lattice [1]. After 35 years, partial results were achieved for equations depending on five points [2]. A lack of progress in this long standing problem is due to the obstruction in the local theory which we are going to overcome by making a proper non-local extension of the associated difference field. The group in Loughborough/Leeds/Kent proposed a resolution of this obstruction [3] which leads to a breakthrough in this important area of research.

The main objectives of the project will be:

1. Study non-local extensions of a difference field, suitable for fractional powers of formal pseudo-difference series and develop symbolic representation for the extended fields and operators.
2. Derive necessary conditions for integrability of semi-discrete dynamical systems and make progress in the problem of classification of integrable cases.
3. Extend the symmetry approach to equations on free associative algebras

[1] R. I. Yamilov. *Russian Math. Surveys*, 38(6):155-156, 1983; *J. Phys. A*, 39:541-623, 2006.

[2] R.N. Garifullin, R.I. Yamilov and D. Levi. *J. Phys. A*, 50(12):125201, 2017; 51(6):065204, 2018.

[3] A.V. Mikhailov, V.S. Novikov, J.P. Wang, Perturbative Symmetry Approach for Differential-Difference equations.