Multidimensional dispersionless equations: geometry and integrability

Supervisors:

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Brief description:

Integrable systems constitute a remarkable class of differential/difference equations which, to some extent, can be `solved explicitly'. This exact solvability is usually due to some internal symmetry which makes integrable systems mathematically rich and interesting objects with deep links to such diverse areas as algebra, analysis, geometry, and mathematical physics. Besides that, integrable systems appear as approximations to more complicated systems of physical origin, suggesting a point of view of integrable systems as new `nonlinear special functions of mathematical physics'. This makes the classifcation of integrable systems an important problem with high potential impact in both pure and applied mathematics.

This project is about a special class of integrable systems known as `dispersionless': such systems govern a broad variety of nonlinear models where dispersive effects can be neglected. Dispersionless equations occur in a wide range of applications in fluid dynamics, averaging theory, general relativity, differential geometry, and the theory of integrable systems. Our group at Loughborough has proposed a novel approach to the study of such equations known as the method of hydrodynamic reductions [1].

In this project we will apply the method of hydrodynamic reductions to the study of higher-order dispersionless equations in three dimensions. The ultimate goal is a complete description of integrable equations within this class. Such equations are expected to have remarkable geometric/symmetry properties, and are of prime importance for the classification of multidimensional dispersive integrable models (which can be efficiently approached based on the method of deformations of hydrodynamic reductions [3]). We will apply the necessary differential-geometric condition for integrability as proposed in [2].

Familiarity with differential equations, differential geometry and computer algebra (Mathematica, Maple) would be a valuable asset for this project.

References:

[1] E.V. Ferapontov, K.R. Khusnutdinova, On integrability of (2+1)-dimensional quasilinear systems, Comm. Math. Phys. **248** (2004) 187-206.

[2] E.V. Ferapontov, K.R. Khusnutdinova, Double waves in multi-dimensional systems of hydrodynamic type: the necessary condition for integrability, Proc. Royal Soc. A **462** (2006) 1197-1219.

[3] E.V. Ferapontov, A. Moro, V.S. Novikov, Integrable equations in 2+1 dimensions: deformations of dispersionless limits, J. Phys. A: Math. Theor. 42 (2009).