Loughborough University

Department of Mathematical Sciences

MATHEMATICAL CHALLENGE

CHRISTMAS - 2018

Problem 1.
Prove that if the equation

$$x^4 + ax^3 + bx + c = 0$$

with real a, b, c has 4 distinct real roots then $ab < 0$.

Problem 2.
Consider the decimal representations of the numbers $x_n = 3^{-n}$, $n = 1, 2, 3, \ldots$:

$$\frac{1}{3} = 0.333\ldots := 0.\overline{3}, \quad \frac{1}{3^2} = 0.\overline{1}, \quad \frac{1}{3^3} = 0.\overline{037}, \quad \frac{1}{3^4} = 0.\overline{012345679}, \ldots$$

where $\overline{a_1 a_2 \ldots a_m}$ is the decimal period.

a) Find the length of the decimal period of x_n.

b) Prove that the decimal period of x_{20} contains the sequence 20182019.

Problem 3.
Newt Scamander and Dumbledore propose to Grindelwald the following bet. Grindelwald can choose any six distinct natural numbers between 1 and 125 and then passes the list to Newt. After that Newt reveals five of these numbers of his choice (one by one) to Dumbledore, and Dumbledore is supposed to guess the remaining sixth number.

Prove that Newt and Dumbledore can always win by proposing a strategy for them.

Can Newt and Dumbledore always win if Grindelwald is allowed to choose numbers between 1 and 500? Justify your answer.

Remarks.

1. There will be a first prize of £50 to the person handing in what will be considered to be the best effort to these problems. There may also be special prizes for the most original solutions.
2. Any student registered on one of the undergraduate programmes in the Department of Mathematical Sciences may submit solutions to any or all of these problems.
3. Solutions should be handed in on or before January 31, 2019 to either Prof. A.P. Veselov (SCH.1.02) or Dr. B. Winn (SCH.M.05), who will be the judges for the Challenge.