# Loughborough University <br> Department of Mathematical Sciences 

## MATHEMATICAL CHALLENGE

## CHRISTMAS - 2015

## Problem 1.

Find a polynomial with integer coefficients such that its minimal value on the real line is $-\sqrt{2}$.
The same question for the minimal value $\sqrt{2}$.

## Problem 2.

For which $n$ and $k$ can one find a sequence $a_{1}, \ldots, a_{n}$ of integer numbers with positive sum, such that the sum of any $k$ consecutive members of the sequence is negative?
Find such a sequence for $n=2015$ and $k=3$.

## Problem 3.

Bond and Le Chiffre play the following game. They have a pile of 36 cards on the table, from which one can take in turn either 2,4 or 7 cards. The player who can not do this is the loser.
For example, if there were only 8 cards left on the table and it is Bond's turn, he can win by taking 7 cards and leaving one card, which Le Chiffre can not take according to the rules.
Le Chiffre is offering Bond to start. Should Bond accept the offer?
The same question for a pile of 52 cards. Justify your answer.
What can you say about the best strategy for any number of cards $n$ and a rule allowing the player to take 2,4 or $2 k+1$ cards for general $n$ and $k$ ?

## Remarks.

1. There will be a first prize of $£ 50$ to the person handing in what will be considered to be the best effort to these problems. There may also be special prizes for the most original solutions.
2. Any student registered on one of the undergraduate programmes in the Department of Mathematical Sciences may submit solutions to any or all of these problems.
3. Solutions should be handed in on or before Friday 29 January 2016 to either Dr. A. Kay (SCH.1.17) or Prof. A.P. Veselov (SCH.1.02), who will be the judges for the Challenge.
