

**Loughborough University**  
**Department of Mathematical Sciences**  
**MATHEMATICAL CHALLENGE**  
**CHRISTMAS - 2017**

**Problem 1.**

Find a non-constant polynomial  $P(x)$  with real coefficients such that

$$P(x)P(2x^2) = P(2x^3 + x).$$

Prove that there exists at most one such polynomial of a given degree and find all of them.

**Problem 2.**

Find two different 3-digit numbers  $A$  and  $B$  in the decimal form such that the 6-digit number  $\overline{AB}$  is divisible by the number  $\overline{BA}$ . (Here for example, for  $A = 201$  and  $B = 735$ , the notation  $\overline{AB}$  means the number 201735.)

For which  $n$  one can find  $n$ -digit numbers  $A$  and  $B$  with the same property?

Justify your answer.

**Problem 3.**

Professor Moriarty promised to stop his agents committing murders if Holmes can win in the following game.

Moriarty puts all 52 cards faces down on the table in a row in any order he wants, and Holmes should find the card of his choice in no more than 26 attempts. Holmes negotiated a deal in which Watson could come before that, see the positions of all cards, do a single swap of two of them and leave without any later communication with Holmes. Moriarty then makes his choice of a card and invites Holmes to find it by opening up to 26 cards. Holmes managed to do this by asking Watson to follow a certain strategy. What could be such a winning strategy? Justify your answer.

*Remarks.*

1. There will be a first prize of £50 to the person handing in what will be considered to be the best effort to these problems. There may also be special prizes for the most original solutions.
2. Any student registered on one of the undergraduate programmes in the Department of Mathematical Sciences may submit solutions to any or all of these problems.
3. Solutions should be handed in on or before January 31, 2018 to either Dr. A. Kay (SCH.1.17) or Prof. A.P. Veselov (SCH.1.02), who will be the judges for the Challenge.