The search for trading partners and the cross-border merger decision.

T.Huw Edwards
And
Ben Ferrett

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T.Huw Edwards* and Ben Ferrett,
Dept of Economics, Loughborough University, Leics LE11 3TU, UK.

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Abstract

We investigate the merger decision between two firms in an outsourcing relationship, one upstream and the other downstream. The inter-firm relationship is subject both to ex ante matching uncertainty and to contractual efficiency issues. Cross-border merger is assumed to solve the latter problem, but at the expense of curtailing the match-searching process. The trade-off between these two factors is assumed to determine the dynamics of foreign direct investment in this kind of industry.

Keywords: Trade, search, outsourcing, merger.

JEL classifications: F12, F23.

*Corresponding author. T.H.Edwards@lboro.ac.uk. (44)1509-2227176.
1 Introduction

Alongside the rapid growth in world trade in recent decades, there has been an even more rapid growth in international trade in intermediate goods. It is becoming increasingly common for goods to be produced by a vertical supply chain that stretches over more than one country – a process known as ‘vertical fragmentation’ (Feenstra, 1998; Hummels et al., 2001; Yi, 2003). In consequence, the share of imported components in spending on final goods has generally been rising. For example, Spencer (2005) reports that, from 1974 to 1993, imports as a proportion of total purchases of electrical equipment and machinery rose from 4.5% to 11.6% in the USA and from 13.2% to 30.9% in Canada.

The drastic growth in vertical fragmentation and the international sourcing of intermediate goods raises the question: How are vertical, cross-border business relationships organised? We analyse the procurement process of a downstream firm in the North that wants to buy components from, or have them processed by, an upstream firm in the South. The downstream Northern firm must choose between two possible structures for its vertical trading relationship: vertical foreign direct investment (FDI), where it merges with a Southern component supplier, and outsourcing, where it trades with a Southern firm through an arm’s length contract.

Figure 1 below, which is taken from Spencer (2005), charts the huge growth of China’s manufacturing exports between 1988 and 2003. In 2003, the majority (57%) of manufacturing exports from China were so-called ‘processing exports’, represented by the sum of the black and grey bars in the figure. ‘Processing exports’ are exports produced using imported inputs, so the processing activity in China forms part of an international supply chain, and the data allow us to analyse how such vertical trade is organised.
The black bars in Figure 1, labelled ‘FIE Processing Export’, are the exports of processed components by foreign-owned enterprises (‘Foreign Invested Enterprises’) in China, and they result from vertical FDI in China. The grey bars, on the other hand, are exports of processed components that result from outsourcing contracts between foreign buyers and independent Chinese firms.

The message of Figure 1 is that vertical ‘processing’ trade with China is increasingly organised through FDI rather than outsourcing – the black bars grow in size over time relative to the grey bars. Our model of the FDI/outsourcing decision is consistent with this stylised fact that vertical FDI grows in importance relative to outsourcing over time.

The baseline version of our model analyses a Northern firm’s search for a trading partner in the South. By paying a fixed search cost, the Northern firm meets a randomly chosen Southern firm. The randomness relates to the quality of the match between the two firms; that is, the total profitability of the vertical trading relationship. Next, the Northern firm must choose how to structure its relationship with the Southern firm. The choice – between merging (vertical FDI) and contracting (outsourcing) – incorporates the key trade-off in our model.

A merger is irreversible (demerger is assumed to be prohibitively costly), but it maximises the value from the vertical trading relationship. In contrast, outsourcing relationships are more flexible (arm’s length contracts last for only one period), but – due to contractual inefficiencies – they waste some of the value in the trading relationship. (The contractual friction might arise from relationship-specific investments that
are only partially contractible.)

After the Northern firm has made its merge/contract choice, output is produced at the end of the first period. Following a merger, the pairing of firms remains together into the infinite future. However, a contract is dissolved after one period, and the Northern firm searches again and repeats the entire process in the next period. We assume that match quality is independently and identically distributed through time. (The informal justification of one-period contracts is as follows. If a pairing wished to stay together for two periods, they would also want to stay together forever because the environment is stationary over time. However, in the case of an infinitely-lived pairing, a merger dominates a contract in profit terms. Therefore, contracts will last only for one period.)

We follow Grossman and Helpman (2002) in adopting a “transactions cost” approach to the integration/outsourcing decision. This is in the tradition of Coase and Williamson, and it views integration as (entirely) resolving contractual problems. (McLaren, 2000, also adopts a ‘transactions cost’ approach.) An alternative approach to analysing contractual frictions is the “property rights” theory of Grossman and Hart, and Hart and Moore. Antràs (2003) analyses the integration/outsourcing decision in this tradition, as do Antràs and Helpman (2004). In the ‘property rights’ approach, vertical integration does not resolve contractual frictions. However, by allocating ‘residual control rights’ over assets (i.e. ownership of assets), integration alters the ‘threat point’ that emerges when the contract breaks down or doesn’t apply.

Whether the ‘transaction cost’ or ‘property rights’ approach is preferred depends upon the exact details of the vertical relationship one has in mind. To the extent that integration entails joint profit maximisation (as merger does) and the acquirer obtains the target’s blueprints (and is able to exploit them as efficiently as the target could), then the ‘transaction cost’ approach (integration resolves contractual frictions) seems appropriate.

We derive the cut-off between contracting and merging by comparing present values. The present values of both merging and contracting are increasing in the match quality that the Northern firm draws. However, the present value of merging is more sensitive to the match-quality draw because, following merger, the pairing of firms remains together forever. Therefore, merger becomes ‘more likely’ as match-quality rises, and we derive a unique cut-off between contracting and merging. Ceteris paribus, we show that contracting
is made more attractive by rises in contract quality, and falls in the discount rate (more patience) and the fixed search cost. These results are intuitive.

Although firms are ex ante identical, pairings that turn out to be of a higher quality are ‘more likely’ to lead to FDI. Moreover, trading relationships organised through vertical FDI earn higher profits than do those organised through outsourcing. This is consistent with the empirical findings of Helpman, Melitz and Yeaple (2004) on the profitability of MNEs relative to other types of firm.

Over time, the likelihood that a firm will merge rises. When a firm enters a new country to source components, outsourcing provides an attractive, flexible means of exploring the market for trading partners. Eventually, after (perhaps) several contractual relationships with different temporary partners, the firm finds a suitable permanent partner for merger. Therefore, in our model, outsourcing is equivalent to “ongoing search”, whereas vertical FDI is chosen by ‘matched’ pairings.

The pattern of Chinese ‘processing exports’ over time in Figure 1 is consistent with our results. The relative importance of vertical FDI grows over time as Northern firms become more familiar with the host country. Accounting for the outsourcing/FDI mix in Figure 1 is an important achievement. For example, Grossman and Helpman (2002) find that thick markets with many potential trading partners favour outsourcing. However, this is not the picture in China. In Figure 1, as China has industrialised since the mid-1980s and its export-oriented manufacturing sector has expanded, we have actually observed a growth in vertical FDI relative to outsourcing. Thus, market thickness appears to be positively correlated with vertical FDI.

We allow for the simultaneous free entry of firms at the start of the search process. Because higher contract quality raises the present value of contracting, host countries with higher contract quality attract more entry by searching firms from the North. Therefore, in the long-run steady state, when firms are matched through vertical FDI, contract quality is positively correlated with national inward FDI intensity. Therefore, the result of the OLI framework (Markusen, 1995) that greater contract quality (e.g. stronger intellectual property rights) favours outsourcing over “internalisation” through FDI is primarily a short-run result. We further extend the model to allow for endogenous contract length, growing markets, and “insider” firms that are better informed than others about search possibilities.
2 A model of interfirm trade with search and matching

We set up a partial equilibrium model of a monopolistically competitive industry in a two-country world - the two countries being the North, \(N\), and the South, \(S\). The main market for final goods is in the North. Production requires two stages, upstream (\(u\)) and downstream (\(d\)). Typically these are carried out by a pairing of firms, which may or may not be vertically integrated by merger. Firm \(u\) sells a semi-finished good to \(d\), who then completes the manufacture and sells it on to final consumers. There are potentially four types of firm pairing - \(NN\), \(NS\), \(SN\) and \(SS\) - where, for example, \(NS\) involves an upstream firm in \(N\) and a downstream firm in \(S\). Due to a technological difference, \(N\) has a potential comparative advantage in producing \(d\), while \(S\) has a potential comparative advantage in \(u\), although firms who wish to produce as \(SN\) pairings need to search for partners, which is assumed to be a costly process.

All firms are of equal size and \textit{ex ante} expected efficiency: however, there is an ongoing fixed coordination cost which varies depending on the goodness of fit of the match, \(\mu_i\). Since match quality, \(\mu\) is an ordinal ranking, it is easy to choose units such that, seen before entering a match, \(\mu\) follows a uniform rectangular probability distribution between 0 and 1, where 1 represents the most beneficial match. Consequently, we can always say that there is an \textit{ex ante} probability \(1 - \mu_i\) of finding a match of better quality than \(\mu_i\).

We focus mainly on the firm pairings of type \(SN\), which benefit from potential comparative advantage. It is assumed that this interfirm trade is a relatively recent development, following trade liberalisation, and that the market for their output is growing. Unlike previous papers (Grossman and Helpman, 2002, Rauch and Casella, 2003, Rauch and Trindade, 2003) we focus on the search process whereby firms find trading partners (as opposed to concentrating on matched pairings, once search has been completed).

In our model, firms carry out this search by trading: more specifically, a firm draws up a contract with a randomly-chosen partner, specifying the price and volume of inter-firm trade over a fixed contract duration (c.f. Spencer, 2005). Only after firms have entered into such a contract can they determine whether the quality of the match is sufficiently good for it to be worthwhile continuing long-term. We term this type of process ‘match-searching’ (to distinguish it from other models, where search is carried out before trade is started). \textit{Figure 1}, below, shows a decision tree for the match-searching model we use in this paper, seen from the viewpoint of a downstream firm in the North.
The match-searching model is a variant of standard search models (Kohn and Shavell, 1974). The most important theoretical feature of these models is that searching players have to choose between sticking with an existing partner or renewing search: there exists an unique switchpoint, $\mu_i$, at which a player is indifferent whether to continue or resume search. In our model, we term this switchpoint the ‘reservation match quality’, $\mu_R$.\textsuperscript{1}

\textbf{Definition 1} The reservation match quality, $\mu_R$, is the match quality at which, for given contract length, contract quality, discount rates and search costs, firms are indifferent whether to continue with their existing partner or to resume search.

In a frictionless search (where firms are infinitely patient) we would expect $\mu_R = 1$, so that firms will never settle for a less than perfect partner. However, we assume search is costly for two reasons, both of which reduce $\mu_R$. First, contracts are ‘lumpy’, being for a fixed length in order to insure firms against potential hold-up problems after they make relationship-specific investments. Some industries may be characterised

\textsuperscript{1}There are parallels to the reservation wage in the labour search literature.
by lumpier contracts than others (Antras, 2003). In combination with impatient firms (discount rate \( r > 0 \))
this leads to search friction. In addition, we assume each renewed search starts with a relationship-specific
investment cost \( E \). In general, following Rauch and Trindade (2003) we assume that this investment is larger
(and implies ‘lumpier’ contracts) the more differentiated goods in an industry are. This is discussed in more
detail in section ___ below, where we endogenise contract length.

Simplifying the model, by making contract periods exogenous, allows us to define a variable for interest
rate per contract period, \( \rho \), where

\[
1 + \rho = (1 + r)^T,
\]

where \( r \) is the annual interest rate and \( T \) is contract length. This is a key factor in determining behaviour
of the search and matching process.

3 Contract length, contract quality and the reservation match quality

Figure 2a, below, shows the determination of the reservation match quality, in the simple version of our model.
Match quality, \( \mu_i \), is defined to vary between 0 and 1 (with constant probability density \( ex \ ante \)).\(^2\) For any
given market price level, \( P^* \), profitability, \( \pi_i \), is monotonically increasing with respect to \( \mu_i \). The reservation
match quality is \( \mu_R \), yielding a profit \( \pi_R(\mu_R) \). This will be equated to the expected present discounted value
of abandoning a current partner and renewing search. Free entry and exit in a monopolistically competitive
model will also usually equate \( \pi_R(\mu_R) = 0 \).\(^3\)

Firms with matchings to the left of \( \mu_R \) will be loss-making, and are termed ‘searching pairings’. Those
to the right will be profitable, and will choose to stick with their existing partner when the contract comes
up for renewal - consequently, we term them ‘matched pairings’. Proportion \( \mu_R \) of pairings are initially
searching, and the average loss incurred by a searching pairing is \( \pi_s = \frac{\text{Area } A}{\mu_R} \). Likewise, proportion \( 1 - \mu_R \)
of initial pairings is initially matched, making an average profit of \( \pi_m = \frac{\text{Area } B}{1 - \mu_R} \).

\(^2\)Specifically, for simplicity, we assume \( \mu_i \) affects an ongoing fixed cost, rather than unit variable costs, so that output of a
firm pairing is not a function of \( \mu_i \). This is fairly easily relaxed.

\(^3\)This relationship holds so long as we are looking at a growing industry, where there is continuous entry of new firms.
At time 0, on average firms are making a loss (area A is bigger than area B). The extent of this loss depends on the position of \( \mu_R \) - the higher \( \mu_R \) is, the more firms will be loss-making in the first period, and the average loss of a searching firm will be higher, and the average profit of matched firms will be less. In addition, the proportion of searching firms declines at a rate of \( (1 - \mu_R) \) for each contract period, so that a high reservation match quality (which implies picky firms) means only slow convergence. Such a process will only be acceptable to firms if \( \rho \) is low, which means either low interest rates, \( r \), or short contract periods.

**Proposition 1** The relationship between area B (the first period profits of successfully-matched firms) and area A (the first period losses of unsuccessfully matched firms) is determined by the formula

\[
\frac{\text{Area } B}{\text{Area } A + E} = \frac{\rho}{1 + \rho}.
\]

(See Appendix 1 for the derivation of equation (2)). A higher value for \( \mu_R \) will lower area B raise area A, implying that \( \rho \) must fall. When \( \rho = 0 \) (infinitely patient search), area B will equal zero.

Figure 2b introduces the idea of inefficient contracts for searching firms (those with \( \mu_i < \mu_R \)). In this case, we are assuming that, where firms have a relationship based on contracted outsourcing, profitability is reduced to \( k\pi_i \), where \( k \ (0 \leq k \leq 1) \) reflects the contractual environment, which differs across countries.
Firms can overcome these contractual problems by merging, but, since we assume demerger is prohibitively expensive, this will only be done once firms have settled on a long-term partner. Consequently, we assume that firms with \( \mu_i \geq \mu_R \) will merge quickly.

Reducing profitability for searching firms will affect the determination of \( \mu_R \). Specifically, area A is reduced as \( k \) falls (unless there is a compensating fall in \( \mu_R \)), which implies that the reservation match quality will be reduced, the lower is \( k \). We can summarise this result as:

**Proposition 2** Any given pairing of firms are more likely to settle for merger the lumpier is contracting, or the poorer the contracting environment, as measured by \( k \).

This relationship is monotonic.

*Figures 3-4* summarise the effects of these trade-offs. First consider a firm which makes its first, random match and finds it is of quality \( \mu_i = m_1 \). Whether this will be an acceptable long-term match depends on combiness the lumpiness of the contract (summarised by \( \rho \), which depends primarily on the industry concerned) and on contract efficiency, \( k \), which is assumed to depend mainly on the countries concerned. The lower is \( \rho \) or the higher is \( k \), the pickier firms can afford to be, and the less likely they are to accept a long-term partner of match quality \( m_1 \). We have drawn in the locus \( \mu_R = m_1 \), which is the combination of \( \{k, \rho\} \) which make the firm indifferent whether to continue with a match of quality \( m_1 \) or not. For combinations of \( \{k, \rho\} \) which lie below this line, the firm will see a match of quality \( m_1 \) as temporary only, and so will choose an outsourcing relationship for the duration of its existing contract. For \( \{k, \rho\} \) above the line, the firms are less fussy, and will choose a long-run merger. Note the locus \( \mu_R = m_1 \) slopes upwards to the right, indicating that there is a trade-off between the levels of \( k \) and \( \rho \) which make firms indifferent whether to merge.
Figure 3: Effects of contract quality and lumpiness on the decision to outsource or merge, given a particular contract quality

Figure 4: Effects of contract quality and lumpiness on reservation match quality

Figure 4, above, develops the relationships in Figure 3 further, by showing as a contour map the combinations of \( \{k, \rho\} \) which correspond to a variety of different levels of \( \mu_R \). As we move downwards to the right (increasing \( k \) or decreasing \( \rho \)) the reservation match quality increases.

### 3.1 Implications of the reservation match quality

The reservation match quality, \( \mu_R \), is the most important parameter in determining search behaviour. A higher rate of \( \mu_R \) implies that firms are pickier in terms of their partners - implying that the search process will take longer, in terms of contract periods. The mean lag (average number of contract periods taken to merge) will equal

\[
T = \frac{\mu_R}{1 - \mu_R}
\]  

(3)

Consequently, a rise in \( \rho \) or a fall in \( k \) will result in a shortening in the number of contracts which firms undergo on average before merging.

Linked to the effects of higher \( \mu_R \) in terms of slower matching, the average ratio of searching to matched firms will tend to be higher, even in the long run, in a growing economy. If firms were infinitely-lived and there were no growth in demand over time, the economy would tend towards a steady-state equilibrium where all firms were matched \( (\mu_i \geq \mu_R) \). However, If the number of firms in the industry were growing at
rate $G$, the long-run ratio of the number of contracting firms, $N_C$, to merged ones, $N_M$, would become:

$$\frac{N_C}{N_M} = \frac{G}{1 - \mu_R}. \quad (4)$$

This implies that,

**Proposition 3** For a given growth rate of demand, the long-run equilibrium share of contracting to merged firms is lower, the lumpier is contracting or the poorer the contracting environment, $k$.

This follows since $\frac{N_C}{N_M}$ is increasing with respect to reservation match quality, $\mu_R$, which, in turn, is decreasing with respect to $\rho$ or $k$.

A third effect is that, if $\rho$ is lower, or $k$ higher, so that the reservation match quality, $\mu_R$, is higher, then the mean match quality in the long run will be higher. This follows since raising $\mu_R$ means that the least efficient matched firms will no longer stick with their existing partners, hence raising the average.

Connected to this is the effect on prices. We define

**Definition 2** The reservation price of a pairing of firms based in countries $c$ and $d$, $P_{Rcd}$, is the minimum level of the market price at which a new pairing would start a search.

It is relatively trivial to show that $P_{Rcd}$ is declining in terms of $C_{cd}$, the underlying variable production cost for the pairing, and declining in terms of $\mu_R$ (since higher $\mu_R$ implies that, in the long run, only pairings of a higher efficiency will survive). As long as the industry continues to grow, so that new firms are still entering, and as long as $SN$ is the pairing with potential comparative advantage, then the industry price will be $P_{RSN}$. A rise in $\mu_R$ implies that new entrants must be prepared to supply at a lower price.

**Proposition 4** The lumpier is contracting, or the poorer the contracting environment, $k$, the higher will be the reservation price of new firm pairings, and the lower will be the long-run efficiency of the industry.

The proof is that $\mu_R$ is decreasing in with respect to $k$ (Proposition 1) or $\rho$. Lower $\mu_R$ implies firms in a reservation quality matching are bearing a higher match-related cost, which must require a higher price to make entry profitable. Also, a lower $\mu_R$ implies lower quality matchings will survive in the longer run - so lowering average quality.
Note that this lower entry price means that, in the early years of search, firms will, on average, bearing a larger loss, as they carry out a longer search. Also that new firm pairings of type SN will be competing with historic pairings NN. Anything which lowers \( \mu_R \) will make new pairings type SN less competitive relative to existing NN pairings. Beyond a threshold, trade may simply not take place: \(^4\) this is likely to be the case in industries with lumpy contracts in countries where the legal/contractual environment is poor. \(^5\)

### 4 Insider versus outsider firms

We have established in the preceding sections that lumpy contracts and/or poor contracting environments tend to favour a faster merger over outsourcing, but beyond a point they may prevent vertical inter-firm trade altogether. We wish to extend the analysis to the situation where not all searching firms are equal. More specifically, we follow Rauch and Trindade’s (2002) analysis, based on the idea that ethnically-based or other trading networks can give a minority of ‘insider’ firms an advantage over others. This approach can be extended to show why ethnic, cultural, linguistic and historic colonial ties regularly appear as significant in gravity analyses of trade. Insider firms start with a better information set than their rivals, so that, instead of match quality \( \mu_i \) varying between 0 and 1, for insiders it varies between \( \mu_L \) and 1 (\( \mu_L > 0 \)).

We apply this approach to our match-searching framework. The model is unchanged from that above, except that there is a subset of ‘insider’ firms for whom the minimum quality match, after they have been through pre-screening, is now of quality \( \mu_L \), rather than 0. This will affect the search process, since it will also lead ‘insider’ firms to ask for a higher reservation match quality, which we denote \( \mu'_R \), before accepting a merger. This process is summarised in Figure 5, below.

\(^4\)These threshold effects in outsourcing trade have been noted, among others, by Yi (2003).
\(^5\)This finding is borne out by Nunn (2007).
Once a minimum quality threshold of $\mu_L$ is introduced, the proportion of searching firms becomes $\frac{\mu_R - \mu_L}{1 - \mu_L}$, which is also the rate of exponential decay in the probability of firms searching as we move from one contract period to the next. Raising the minimum quality threshold to $\mu_L$ will initially lower the expected loss of searching firms, as well as lowering the proportion of searching firms. A corollary of this is

**Proposition 5** where the underlying cost advantage of the South is relatively small, only insider firms from the North will enter as potential partners.

This follows since pairings of an insider Northern firm with a Southern partner have better average quality, and so require a lower reservation price $P''_{RSN}$ than that of outsider pairings $P_{RSN}$. It follows that there must be levels of cost for $NN$ pairings such that $P''_{RSN} < P_{RNN} < P_{RSN}$.

This proposition must be slightly qualified in the longer run: search may make new pairings of an outsider firm in $N$ with a southern partner uncompetitive relative to existing $NN$ pairings or insider $SN$ pairings, but if the supply of these latter firms is limited, and if demand is growing, then in the long run the market price must rise to the point where outsider $SN$ pairings begin to compete.
5 The effect of making contract length endogenous

We have deliberately separated the endogenous determination of contract periods from the analysis so far in the paper. This is because, while there are a number of practical reasons to believe that factors such as relationship-specific costs and the contractual environment will affect the length of contract periods, as well as the reservation match quality, the exact theoretical mechanism is harder to pin down.

We set up a stylised game where firm \( u \) faces a choice of whether or not to spend \( E \), which cannot be verified by firm \( d \). If firm \( u \) makes the investment, it will save a random amount \( \frac{k\mu}{2} \) or \( \frac{\mu^2}{2} \) (depending on whether the match is good enough to justify merger) per annum over the period of the initial contract. \( d \) will save a similar amount. If firm \( u \) does not make the investment, then all firm \( d \) will perceive is a very low observed level of \( \mu_i \). Consequently, the contract needs to be long enough that, on average, firm \( u \) expects to be better off risking the relationship-specific investment rather than deceiving its partner. This is shown in Figures 6a-b, below.

Looking at Figure 6a first: this is the same as Figure 2a, except that we have drawn in an Area C, equivalent to

\[
\text{Area C} = \int_{\mu_i=0}^{1} (\pi(\mu_i) - \pi(\mu_0))d_i,
\]  

(5)
in other words, the expected average difference in profit (across all potential matches) relative to the worst potential match. This is the difference between the profit (per annum) the firm gets if it makes the investment \( E \), compared to if it deceives its partner. Contract length, \( T \) will be sufficient to yield

\[
\frac{\rho}{1 + \rho} = \frac{rE}{\text{Area } C};
\]

\[
\frac{\text{Area } B}{\text{Area } + E} = \frac{rE}{\text{Area } C}. \quad \text{Substituting from (2)}
\]  

Figure 6b shows the effect of lowering contract efficiency, \( k \), for searching firms. In this case, area C is reduced, implying \( \rho \) must be higher and contracts must be longer. In consequence, there are now two factors reducing \( \mu_R \): the lower profit (as before) and the longer search period. A conclusion is

**Proposition 6** if contract periods are endogenous, then the effects of contract efficiency upon reservation match quality, mean merger lags and reservation prices are enhanced, compared to the model with exogenous contract periods.

This follows since, if raising \( k \) now affects \( \mu_R \) both directly and through raising \( \rho \), then its effects on variables which depend on \( \mu_R \) will be enhanced. In summary, the effects of changing variables in equation (6) are as follows:

<table>
<thead>
<tr>
<th>Effect of/on</th>
<th>( \frac{B}{\text{Area } + E} )</th>
<th>( \rho )</th>
<th>( T )</th>
<th>( \mu_R )</th>
<th>( P_R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raising ( r )</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Raising ( E )</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Lowering ( k )</td>
<td>+</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

6 A more formal model

The preceding sections have aimed at giving qualitative insight into the effects of contract lumpiness, contractual efficiency and insider-outsider differences in a search-based trade model. However, to quantify these effects, and gauge their importance, we need to specify more specific functional forms. This involves formally laying out models of the competitive structure, the matching process and the effects of institutional parameters.
6.1 Competitive structure

The industry is assumed to be monopolistically competitive, on the lines of Krugman (1979). There are both fixed and sunk costs (in the form of a relationship-specific investment, $E$). Subject to these, firms can enter or exit the market, although they need a partner (existing or new) in order to produce saleable goods. The elasticity of substitution between final goods varieties is $\varepsilon > 1$, which closely approximates the own-price elasticity for the output sold by firm pairings, at least as long as the number of firms, $N$, is large.

Since the love-of-variety model is fairly standard, important equations are summarised in Table 1, below.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Important equations of the monopolistically-competitive model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility</td>
<td>$U = A^{\frac{1}{\varepsilon}} \left( \sum_{i=1}^{N} Y_i^{\frac{1}{\varepsilon}} \right)^{\frac{1}{1-\varepsilon}}$. (7)</td>
</tr>
<tr>
<td>Price charged by pairing $i$</td>
<td>$P_i = \frac{\varepsilon}{\varepsilon - 1} C_i, \ i \in {N, S}$. (8)</td>
</tr>
<tr>
<td>Output of pairing $i$</td>
<td>$Y_i = A(\frac{C_i}{(\varepsilon - 1)P_i})^{-\varepsilon}$. (9)</td>
</tr>
<tr>
<td>Industry aggregate price</td>
<td>$P^* = A^{\frac{1}{\varepsilon}} \left( \sum_{i=1}^{N} P_i^{\frac{1}{\varepsilon}} \right)^{\frac{\varepsilon - 1}{\varepsilon}}$. (10)</td>
</tr>
<tr>
<td>Profit of pairing before fixed cost</td>
<td>$\pi_i = \Phi C_i^{1-\varepsilon} P^{\varepsilon}, \text{ where } \Phi = \frac{A}{\varepsilon}(\frac{\varepsilon}{\varepsilon - 1})^{1-\varepsilon}$. (11)</td>
</tr>
<tr>
<td>Profit after fixed cost (merged pairing)</td>
<td>$\Pi_i = \Phi C_i^{1-\varepsilon} P^{\varepsilon} - F + \mu_i, \text{ where } F \geq 1$. (12a)</td>
</tr>
<tr>
<td>Profit after fixed cost (unmerged pairing)</td>
<td>$\Pi_i = \Phi C_i^{1-\varepsilon} P^{\varepsilon} - F + k_0, \text{ where } 0 \leq k \leq 1$. (12b)</td>
</tr>
</tbody>
</table>

Following Grossman and Helpman (2002), we assume matching affects fixed rather than variable cost - this is done primarily to make the model more tractible.\(^7\) More specifically, there is an ongoing fixed cost of between 0 and 1, which is inversely linearly related to the quality of the match $\mu_i$ between firms in pairing $i$. Assume that fixed costs are $F - \mu_i$ for merged pairings and $F - k_0$ for unmerged pairings.

We assume that merger is forever - if two firms merge, then any subsequent demerger would entail prohibitive costs.

---

\(^6\) The restriction $\varepsilon > 1$ is associated with consumers’ assumed ‘love of variety’, and also helps ensure finite pricing by firms.

\(^7\) The attraction of assuming that matching affects fixed, rather than variable costs, is that match quality affects only profitability, not price or sales. This is somewhat at the expense of realism, but improves the tractibility of the model. It is also possible to set up the model where variable cost, $C_i$, is a function of $\mu_i$; results are available from the authors. The principal conclusions of this paper are unaffected.

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6.2 Derivation of the reservation match quality

The *ex ante* probability distribution for match quality (for outsider firms) is uniform and rectangular over the range $0 \leq \mu_i \leq 1$. The profit of a reservation quality match ($\mu_i = \mu_R$) is zero. Profit increases linearly with respect to match quality, so that the expected profit of a successful match (where $\mu_R < \mu_i < 1$) is $\frac{1+\mu_R}{2}$ minus the profit of a reservation quality match. This yields an expected profit of $\frac{1-\mu_R}{2}$. Likewise, an unsatisfactory match will have an expected profit of $-\frac{\mu_R}{2}$. Finally note that a renewed search will yield a present discounted value of zero.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Probability</th>
<th>Expected profit</th>
<th>First period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>above reservation</td>
<td>profits/losses for pairings</td>
</tr>
<tr>
<td>Satisfactory</td>
<td>$(1 - \mu_R)$</td>
<td>$\frac{1+\mu_R}{2} - \mu_R = \frac{1-\mu_R}{2}$</td>
<td>Area B profits of successful $(1 - \mu_R)^2$</td>
</tr>
<tr>
<td>Unsatisfactory</td>
<td>$\mu_R$</td>
<td>$\frac{k\mu_R}{2} - \mu_R = \frac{(k-2)\mu_R}{2}$</td>
<td>Area A losses of successful $(2 - k)\mu_R^2$</td>
</tr>
</tbody>
</table>

Both areas on *Figure 2* are quadratic expressions in terms of $\mu_R$, so it should be no surprise that, in this model where profits are a linear function of match quality, the relationship in equation (2) reduces to a quadratic expression in $\mu_R$. Solving this, and taking the only feasible root ($0 \leq \mu_R \leq 1$), we derive a relationship between $\mu_R, \rho, k$ and $E$ (see Appendix 1 for derivation)

$$\mu_R = \frac{1 + \rho}{1 - (1-k)\rho} - \frac{\sqrt{(1+\rho)(2-k)\rho + \rho E(1 - (1-k)\rho)}}{1 - (1-k)\rho}.$$  \hspace{1cm} (13)

Note that we would expect a rise in $\rho, k$ or $E$ to reduce $\mu_R$. When $E = 0$ and $k = 1$, (13) simplifies to

$$\mu_R = 1 + \rho - \sqrt{\rho(1+\rho)}.$$ \hspace{1cm} (13a)

6.3 The threshold price

We assume that, prior to a reduction of trade costs, all production is by pairings $NN$, at a marginal cost of $C_{NN}$ and all charging a markup as in equation (8). Since monopolistic competition ensures that a marginal
pairing of reservation quality will break even, we can deduce by rearranging (11) that

$$P_0^* = \left[ \frac{F - \mu_{RN}}{\Phi} \right]^\frac{1}{\nu} C_{NN}^{\frac{\nu}{1 - \nu}},$$

(14)

where $\mu_{RN}$ is the reservation match quality for a pairing of northern firms, which may differ from $\mu_{RS}$ if the contractual environment in the North and South differs.

Following liberalisation, the marginal pairing is now type $SN$, producing at a combined marginal cost of $C_{SN}$, so that monopolistic competition will drive the industry price, $P^*$, to equal

$$P^*_1 = \left[ \frac{F - \mu_{RS}}{\Phi} \right]^\frac{1}{\nu} C_{SN}^{\frac{\nu}{1 - \nu}},$$

so that

$$\frac{P^*_1}{P_0^*} = \left[ \frac{F - \mu_{RS}}{F - \mu_{RN}} \right]^\frac{1}{\nu} \left( \frac{C_{SN}}{C_{NN}} \right)^{\frac{\nu}{1 - \nu}}.$$  

(15a)

Note that existing pairings in the North of $NN$ will vary in efficiency between $\mu_{RN}$ and 1, and that some of these may still remain profitable even after trade is liberalised.

### 6.4 Insider firms

In the case of insider firms, Table 2 can be rewritten as:

<table>
<thead>
<tr>
<th>Table 3 insider firm</th>
<th>Probability expected profit first period</th>
<th>Match</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>above reservation</td>
<td></td>
</tr>
<tr>
<td>Satisfactory</td>
<td>$\frac{1 - \mu_R^k}{1 - \mu_L}$</td>
<td>$\frac{1 + \mu_R^k - \mu_R^a}{2}$</td>
</tr>
<tr>
<td>Unsatisfactory</td>
<td>$\frac{\mu_R^a - \mu_L}{1 - \mu_L}$</td>
<td>$\frac{k(\mu_R^a + \mu_L)}{2} - \mu_R^a$</td>
</tr>
</tbody>
</table>

Again, substituting into equation (2) produces a quadratic expression for $\mu'_R$, which can be solved to yield

$$\mu'_R = \frac{(1 + \rho) - \rho \mu_L}{1 + \rho(k - 1)} - \frac{\sqrt{((1 + \rho) - \rho \mu_L)^2 + (1 + \rho(k - 1))(k \rho \mu_L^2 - (1 + \rho) + \rho(1 + \mu_L)E)}}{1 + \rho(k - 1)}.$$  

(16)

When $\mu_L = 0$, (23) reverts to equation (12).
A minimum threshold match for insiders, $\mu_L$, reduces the risk of search. Consequently, insider firms will have a higher reservation match quality, $\mu_R^\prime$, than outsiders, and $\frac{\partial \mu_R^\prime}{\partial \mu_L} > 0$, so the reservation match quality rises the better-placed the insider firm is.

### 6.5 Endogenous contract periods

We wish to formalise the relationship in equation (6). In our linear model, area C, the expected saving per annum for the upstream firm from having a match quality greater than zero, will satisfy

$$\text{Area } C = \frac{1 - (1 - k)\mu_R^2}{4}. \tag{17}$$

(See Appendix 1 for the derivation). The contract period must be just long enough that these savings compensate firm $u$ for the expense of making a relationship-specific investment, $E$. Consequently, (16) becomes

$$\frac{\rho}{1 + \rho} = \frac{4rE}{1 - (1 - k)\mu_R^2} = \frac{1 - \mu_R^2}{(2 - k)\mu_R^2 + E}. \tag{18}$$

Note that, the shorter is the contract period, $T$, the lower is $\rho$, and the larger is $\frac{(1+\rho)}{\rho}$. Consequently, for a given interest rate, $r$ and a given relationship-specific investment, $E$, the shorter is $T$, the higher $s$ needs to be.

Solving (18) fully requires solving a highly complicated equation for $\rho$ in terms of $r$, $k$ and $E$, after substituting in for $\mu_R$ from (13) (see Appendix 1). In practice, this is best solved numerically. Results are shown in Figures 7a-b, Appendix 2, for a variety of combinations of $k$ and $E$, in the case where $r = 0.05$. These confirm that higher contract quality implies shorter contracts and a higher reservation match quality, while higher relationship-specific costs lengthen contracts and lower reservation match quality. This analysis confirms that, at least on one possible account of contract length determination, the effects of contract quality on reservation match quality are enhanced, because a poor contracting environment leads to longer contracts, raising $\rho$ and lowering $\mu_R$. Figure 8, which shows that, once endogenous contract periods are taken into account, the mean lag in firms merging increases sharply with $k$, at least when $E = 0.6$. Interestingly, the mean merger lag in years is increasing with respect to the relationship-specific cost, $E$: this is because
the effect of increasing the contract period length, \( T \), outweighs that of reducing \( \mu_R \).

7 A numerical example

In order to illustrate the effects of contractual environment, contract lumpiness and insider/outsider discrepancies, we develop a numerical example. A product market is assumed to have been liberalised a few years ago, and to be growing rapidly. The South is assumed to have a potential comparative advantage (before taking account of search costs) in the upstream stage of production, while the North has an advantage in downstream production, giving a slight potential competitive advantage to \( SN \) pairings: unit variable cost is assumed to be 92.5% of that of \( NN \) pairings. We assume the following parameter values:

<table>
<thead>
<tr>
<th>Table 4</th>
<th>NN pairing</th>
<th>Outsider SN pairing</th>
<th>Insider SN pairing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit variable cost</td>
<td>see text</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Interest rate, ( r )</td>
<td>5%</td>
<td>5%</td>
<td>5%</td>
</tr>
<tr>
<td>Basic fixed cost, ( F )</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Minimum match quality</td>
<td>0.3</td>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>Contract quality</td>
<td>0.9</td>
<td>varies</td>
<td>varies</td>
</tr>
<tr>
<td>Rel-specific cost</td>
<td>varies</td>
<td>varies</td>
<td>varies</td>
</tr>
</tbody>
</table>

The issue is, assuming this market is growing (so new pairings are entering), how big does the cost advantage have to be for the entering pairings to be \( SN \), rather than \( NN \)?

New pairings in the North have two potential advantages. First, all \( NN \) pairings have insider advantage (represented in our simulation as a minimum quality threshold \( \mu L = 0.3 \)). Only a small number of Northern firms have a comparable insider advantage when dealing with potential Southern partners (these are classed as insider \( SN \) pairings). The second advantage is that the contractual environment in the North is assumed to be relatively good, giving a contract efficiency of \( k = 0.9 \) for unmerged pairings. (This setup still gives some advantage to firms which merge.)

Given these potential factors favouring Northerners, we wish to plot, as a function of relationship-specific cost, \( E \), and contract quality in the South, \( k \), whether or not \( SN \) pairings are competitive with new \( NN \) pairings. This is shown in Appendix 2. Figure 7, below, also summarises the effects of \( E \) and \( k \) on the
The first implication of Figure 7 is that poor contract efficiency in the South deters SN pairings from entering, particularly when they are outsiders (rather than the small pool of insider firms). In our simulation where $F = 1$ this is particularly the case where relationship-specific costs are low - however, when $F$ is higher, this is reversed (see Appendix 2). We therefore get three zones in Figure 7: in the first, where $k$ in the South is low, no SN pairings at all enter, and production remains entirely in the North. Moving up slightly from this, insider SN pairings only will enter: these might, for example, be old colonial trading firms, or maybe ethnic Chinese or other relevant ethnic networks. With their better local knowledge, these can reduce somewhat the effects of poor contractual environment. However, these firms are assumed to be limited in supply, so once they have entered, new trading entry may dry up.

In our example (where underlying comparative advantage is not enormous), outsider SN pairings will mostly enter when $k$ is high (so they will concentrate in Southern countries with relatively good contracting environment - as in Nunn, 2007). These firms will enter first as outsourcers, but will merge over time. However, the share of outsourcing may remain significant even in the long run if market growth is substantial, and if $\mu_R$ is high - hence, outsourcing is likely to be more prevalent in the long term where $k$ is high and $E$
8 Conclusion

In common with a number of previous studies, we have examined the role of search in the choice between an outsourcing relationship or vertical FDI. The main difference is that search is seen as an ongoing process, with aspects of learning-by-doing, and the value of outsourcing is its relative flexibility, while the main value of FDI is to reduce transational costs, which in turn vary according to the national legal/institutional environment. As such, our analysis is in the tradition of Nunn (2007) in emphasising the importance of legal/institutional quality as a determinant of trade, at least within differentiated industries with high relationship-specific costs.

A key feature of our model is that it differentiates static and dynamic results. Poor institutional quality is particularly costly to outsourcers. On the one hand, it may deter search altogether, by raising the reservation price of searching firm pairings in a particular market. However, if underlying comparative advantage is strong, then this may offset the increased search costs, so that firms still engage in search. In this case, the FDI decision will be sped up - consequently, we would agree in the short-run with Markusen’s (1995) OLI finding that poor contract quality favours FDI, although adding the caution that this does not apply in a long-run steady-state, where all firms will merge in our framework.

A poor institutional environment in a particular country may limit trade to a relatively small number of firms with pre-existing ties (as in Rauch and Trindade, 2002). Again, these are likely to engage in FDI.

The dynamics of our model indicate that, as a new exporter grows in size, outsourcing will tend to precede FDI. This is somewhat contradictory to Grossman and Helpman’s (2002) prediction (based on a thin markets model with search before trading) that increasing market size will imply an increasing share of outsourcing. We argue that the Chinese experience tends to support our predictions, and perhaps supports the existence of a search-by-matching process.
References


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Appendix: algebraic derivation of the reservation match quality

General functional form

The solution should satisfy the condition

\[ \text{Discounted profit with a successful match = one period expected loss with a poor match + sunk cost.} \]

Expected average joint profit with a successful match is \( \pi_m \), which occurs with probability \( (1 - \mu_R) \) and lasts forever. Discounted to the start of the first contract period, this has the value

\[
\frac{(1 + \rho)}{\rho} (1 - \mu_R) \phi \pi_m,
\]

where \( \phi (r, T) \) is an adjustment factor for contract period length.

The expected loss with a poor match is \( -\pi_s \), which occurs with a probability \( \mu_R \), and lasts for 1 period only. Discounted to the start time, this has present value

\[-\mu_R \phi \pi_s,\]

while there is also a one-off fixed cost, \( E \), related to the search. Equating the present expected value of starting a search to zero, this gives us the result

\[
\frac{(1 + \rho)}{\rho} (1 - \mu_R) \phi \pi_m = \mu_R \phi \pi_s + E.
\]

However, we also note that

\[
\phi = \frac{1 + \rho}{\rho} \left( 1 - \frac{1}{1 + \rho} \right) = 1.
\]

Also note that, in Figure 2a), area B = \( (1 - \mu_R) \pi_m \) and area A = \( \mu_R \pi \). Hence we get the relationship
\[
\frac{(1 + \rho)}{\rho} \text{Area B} = \text{Area A} + E;
\]
\[
\frac{\text{Area B}}{\text{Area A} + E} = \frac{\rho}{1 + \rho}.
\]  

(2)

**Linear functional form**

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Probability</th>
<th>Expected profit</th>
<th>First period</th>
</tr>
</thead>
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<td></td>
<td></td>
<td>above reservation</td>
<td>profits/losses for pairings</td>
</tr>
<tr>
<td><strong>Satisfactory</strong></td>
<td>(1 - \mu_R)</td>
<td>(\frac{1 + \mu_R}{2} - \mu_R = \frac{1 - \mu_R}{2})</td>
<td>Area B profits of successful ((1 - \mu_R)^2)</td>
</tr>
<tr>
<td><strong>Unsatisfactory</strong></td>
<td>(\mu_R)</td>
<td>(\frac{k\mu_R}{2} - \mu_R = \frac{(k-2)\mu_R}{2})</td>
<td>Area A losses of successful ((2 - k)\mu_R^2)</td>
</tr>
</tbody>
</table>

Looking at the decision from the point-of-view of a pairing deciding whether to renew search

\[
1 + \rho (1 - \mu_R)^2 = (2 - k)\mu_R^2 + E;
\]
\[
(1 + \rho)(1 - 2\mu_R + \mu_R^2) = \rho(2 - k)\mu_R^2 + \rho E;
\]
\[
(1 + \rho - \rho(2 - k))\mu_R^2 - 2(1 + \rho)\mu_R + (1 + \rho) - \rho E = 0;
\]
\[
(1 + (k - 1)\rho)\mu_R^2 - 2(1 + \rho)\mu_R + (1 + \rho) - \rho E = 0;
\]

\[
\mu_R = \frac{1 + \rho \pm \sqrt{(1 + \rho)^2 - (1 + (k - 1)\rho)((1 + \rho) - \rho E)}}{1 + (k - 1)\rho};
\]
\[
= \frac{1 + \rho}{1 + (k - 1)\rho} - \frac{\sqrt{(1 + \rho)(1 + \rho - (1 + (k - 1)\rho)) + \rho E(1 + (k - 1)\rho)}}{1 + (k - 1)\rho};
\]
\[
= \frac{1 + \rho}{1 - (1 - k)\rho} - \frac{\sqrt{(1 + \rho)(2 - k)\rho + \rho E(1 - (1 - k)\rho))}}{1 - (1 - k)\rho}.
\]  

(13)

**Endogenisation of the contract period**

To formalise this condition: say the expected rate of saving per annum to firm \(u\) if it makes the investment
is the probability-weighted sum of the saving if it does or does not merge:

$$s = \frac{1}{2}[(1 - \mu_R)\frac{1 + \mu_R}{2} + \mu_R\frac{k\mu_R}{2}] = \frac{1 - (1 - k)\mu_R^2}{4}. \quad (17)$$

Then the contract period must be just long enough that a flow of payments at rate $s$ are just enough to compensate firm $u$ for the expense of making a relationship-specific investment, $E$:

$$E = s \int_{t=0}^{T} e^{-rt} \partial t = \frac{s}{r}[1 - (1 + r)^{-T}],$$

where $r$ is the annual interest rate. Note that

$$1 - (1 + r)^{-T} = \frac{\rho}{1 + \rho}.$$ 

Consequently,

$$s = \frac{(1 + \rho)}{\rho}rE; \quad 1 - (1 - k)\mu_R^2 \frac{4}{4} = \frac{(1 + \rho)}{\rho}rE. \quad (18)$$

Note that, the shorter is the contract period, $T$, the lower is $\rho$, and the larger is $\frac{(1 + \rho)}{\rho}$. Consequently, for a given interest rate, $r$ and a given relationship-specific investment, $E$, the shorter is $T$, the higher $s$ needs to be.
Appendix 2: numerical simulations of making contract periods endogenous

Figures 7: Effect of contract quality and relationship-specific cost on
a) Contract length
b) Reservation match quality

assuming 5% per annum interest rate

Figure 8: Effect of contract quality and relationship-specific cost on mean merger lag in years

Calculation of the breakeven cost level for a NS pairing (insider or outsider)
Start with a modification of (15a)

\[
\frac{P_{SN}^*}{P_{NN}^*} = \left( \frac{F - \mu_{RS}}{F - \mu_{RN}} \right)^{\frac{1}{\varepsilon}} \left( \frac{C_{SN}}{C_{NN}} \right)^{\frac{\varepsilon-1}{\varepsilon}}.
\]

For the breakeven price, we have

\[
\frac{P_{SN}^*}{P_{SN}^*} = 1,
\]

so

\[
\left( \frac{C_{SN}}{C_{NN}} \right)^{\frac{1}{\varepsilon}} = \left( \frac{F - \mu_{RS}}{F - \mu_{RN}} \right)^{\frac{1}{\varepsilon}}; \quad \frac{C_{SN}}{C_{NN}} = \left( \frac{F - \mu_{RS}}{F - \mu_{RN}} \right)^{\frac{1}{\varepsilon}}.
\]

Figures 7a-b: Relative entry prices of outsider and insider pairings, related to contract quality and relationship-specific costs