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R&D Productivity and Intellectual Property Rights Protection Regimes

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Abstract
We study firms’ preferences towards intellectual property rights (IPR) regimes in a North-South context, using a simple duopoly model where a ‘North’ and a ‘South’ firm compete in a third market. Unlike other contributions in this field, we explicitly introduce the South’s capability to undertake cost-reducing R&D, but maintain the South’s inferiority in utilizing and managing its R&D. In contrast to traditional results, we show that the North may encourage lax IPR protection provided that its South rival’s R&D productivity is sufficiently high, while the South may find it in its best interest to strictly enforce IPR protection if its R&D productivity is low. In this sense, our results do not support the idea of universal or uniform IPR protection regime. In addition, we find that if firms are allowed to agree on any level of information exchange when IPR protection is strictly enforced, such an exchange can always be established as long as each firm is ensured that what it gets to utilize in return is greater than a half of what it gives to its rival.

Keywords: intellectual property rights (IPRs), cost-reducing R&D, R&D productivity, information exchange.

JEL Class.: O34, F13, O32, O38, L13, D43.

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1 Introduction

The Asian Tigers, namely South Korea and Taiwan, are examples of countries that emerged as aggressive competitors in consumer electronics, microelectronics, robotics, computers and peripherals, as well as in various services during the 1980s. The erosion of the technological leadership of firms in industrialized countries, notably the United States, in these hi-tech areas has been partially attributed to the too open technological and scientific system which allowed foreign countries to imitate and profit from US innovations (Correa (1994)). This was one of the reasons for the US to aggressively push for a reform in the Intellectual Property Rights (IPR) regimes. Henceforth, the Agreement on Trade-Related Aspects of Intellectual Property Rights (hereafter the TRIPS agreement) was established with the clear objective of universally harmonizing standards of IPR protection. Developing countries reluctantly negotiated increased standards of protection, even though they regarded the TRIPS agreement as a policy of "technological protectionism" whereby the developed countries generate innovations and the developing countries provide markets for the resulting products or services (Correa (2000)).

The TRIPS issue was very critical between the Northern and the Southern countries during the Uruguay round, leading to a North-South confrontation, where the North traditionally refers to countries where inventions or innovations take place, and the South comprises developing countries or countries that are to a large extent dependent upon the innovations made in the North. Innovations by the North can be copied at very low cost by the South without consent from the innovators, so that the strengthening of IPR protection inevitably stirred the conflict of interest between the North and the South.

However, the increasing capacity to innovate in the R&D intensive industries of countries such as South Korea and Taiwan has challenged traditional views. Although these developing countries have followed and relied extensively on the adaptation and improvement of imported technologies in their path of industrialization, some of them have now reached technological levels that could be further enhanced by their own R&D efforts. Some developed minor product and process innovations to be used in their domestic industries (Correa (2000)). For example, South Korea has emerged as a world class competitor in the semiconductor industry while Taiwan has also developed significant capacity in this field. The technological advances in this industry evolve as an interactive, cumulative process where improvement is directly based on the pre-existing stock of knowledge, hence access to the most up-to-date information which may be possessed by the rival firms is very beneficial. Similarly, innovation in software development is typically incremental. It is a process that builds on and interacts with many other features of existing technology to create a new technology. In the case of multimedia products, the re-use of existing copyrighted materials from numerous rights-holders may pose a great burden and entail high transaction costs. Hence for these types of industries strict enforcement of IPR through various types of legal instruments may negatively affect the diffusion of computer programs, the invention of integrated circuits, and related product and process innovations.

To the best of our knowledge, most economic analysis on the impact of the TRIPS agreement on developing countries has not yet reflected their significant technological development (e.g., Chin and Grossman (1990), Diwan and Rodrik...
(1991), Doardorff (1992) and Žigić (1997)). It is generally assumed that innovative activities are solely performed in the North. A South firm does no R&D and just imitates technology produced in the North via spillovers. Chin and Grossman (1990) are the first to formally model the conflict of interest regarding the degree of IPR protection between the North and the South. They show that the South always prefers no IPR protection unless its share of consumption is so high that the Southern consumers gain significantly from the fruits of the North firms’ R&D effort. However, from the North’s point of view, it is always beneficial having its IPR protected by the South regardless of market structure. Hence, there generally exists a conflict of interest between the North and South. Žigić (1998) complements Chin and Grossman (1990) by endogenizing the strength of IPR protection, reflected in the intensity of spillovers, and finds that the final market structure not only depends on R&D efficiency but also on the strength of IPR protection. Diwan and Rodrik (1991) allow for different preferences for technology or products between the North and the South such that markets are segmented as well as for a gradation of IPR protection. Their basic idea is that R&D resources in the North are limited, so choices have to be made as to which area of technology would receive greater emphasis. They find that an increase in the IPR protection in either of the two regions increases the North’s innovative activities, and IPR protection in one region can skew the technology range away from the needs of the other region. So when the difference in preference is substantial, the South may benefit by strengthening its IPR protection so that it can influence the choice of technology or product developed by the North.

Lai and Qiu (2003) are among the first to assume that both the North and the South have innovative capability in conducting product innovation. To address the issue of multi-sectoral (multi-issue) negotiations in the GATT/WTO context through a bargaining game, they assume two types of goods produced in each region, the differentiated and traditional products. Innovation and imitation are carried out only in the differentiated product sector, thus IPR protection is an issue in this sector. On the other hand, tariff reduction granted by the North to the imported traditional good produced by the South is used as an incentive for the South to protect the North’s IPR. They show that without appropriate tariff concession given by the North, the South would find strengthening its IPR protection makes it worse off, and has no incentive to harmonize its IPR protection with the North’s.

In contrast to the papers mentioned above, we allow for process innovation in the South and for the firms to benefit from each other’s innovation under lax IPR protection, when spillovers abound. In this aspect, our basic setup falls into the typical “R&D with spillovers” models. When IPR protection is lax spillovers are high and refer to leakages in technological know-how so that each firm’s final cost reduction is the sum of its autonomously acquired part and a fraction of the other firms’ part where such fraction indicates the intensity of

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1Grossman and Lai (2004) also study the incentives that governments have in choosing the level of optimal patent length in a North-South context. In an open economy setting, they assume that although both countries have innovating firms, the North has a greater innovative capacity. They show that with national treatment of IPR protection being applied, the differences in market size and differences in capacity for R&D cause differences in country’s optimal patent policy.

the R&D spillovers.

One important feature we introduce is the asymmetry in R&D productivity between the 'North' and the 'South' firm. With a long experience in R&D activity, the North is naturally perceived as being more R&D productive than the South. Firms possess different levels of ability to absorb and utilize R&D output, as well as different levels of cost efficiency. Factors governing these differences range from different organizational and managerial structures to the individual talent of engineers and R&D experts. One form of R&D management may allow or force R&D personnel to extract benefits or utilize the fruits of R&D activities more productively than others. In a theoretical sense, these differences can be portrayed by different levels of pre-innovation marginal cost, R&D productivity, or R&D cost efficiency.\(^3\) We define our R&D productivity parameter in the same way as Barros and Nilssen (1999), that is: the rate at which R&D activities transform into cost reduction. Our present model concentrates on the asymmetry of R&D productivity.\(^4\)

We aim to examine the importance of the asymmetry in R&D productivity and how this affects firms' perceptions toward the strength of IPR protection. We look at a possible conflict of interest between the North and the South regarding the appropriate IPR protection regime and explore under what circumstances such a conflict could be avoided. In doing so, we propose a model embodying both asymmetry in firms' R&D productivity and spillovers and consider how different degrees of IPR protection regimes affect the North and the South firms' choices of R&D as well as their profits.

We show that in spite of being a superior innovator, the North does not always benefit from having its IPR respected; this depends on the level of the South firm's R&D productivity. If the South's productivity is high enough both firms benefit from high spillovers and hence would prefer a lax IPR regime. Moreover, when the South firm realizes that its R&D productivity is much inferior compared to the North firm, the South firm prefers strict IPR protection. In addition, if firms are allowed to engage in information exchange when IPRs are fully protected, such info-sharing agreement can always be established as long as each firm knows that what it can get and utilize is sufficiently more than what it gives to its rival.

In the following section, we present the basic model and calculate the equilibrium under two IPR protection regimes: lax and strict; we then compare R&D, output and profit across regimes. In section 3 we discuss the case where the firms are allowed to use side payments to press for a particular type of IPR protection. In section 4, we extend the model to allow firms to decide whether they would want to engage in information exchange when governments enforce IPR protection strictly. Finally, in section 5 we provide some concluding remarks.

\(^3\)R&D productivity refers to ability to bring down the marginal cost of production, while R&D cost efficiency concentrates on the cost of conducting R&D.

\(^4\)Poyago-Theotoky (1996) considers the firms' incentive to invest in the situation where spillovers are absent and the competing firms start off with different pre-innovation marginal costs.
2 The Model

Two firms, designated North, \( (n) \), and South, \( (s) \), engage in cost-reducing R&D and export all their (homogenous) product to a third market with linear inverse demand \( P = A - \sum_{i=n,s} q_i \), where \( P, A \) and \( q_i \) denotes price, size of the market and firm \( i \)'s quantity respectively. For simplicity, we normalize the size of the market to 1.

In the absence of R&D, both \( s \) and \( n \) produce with the same production marginal cost, \( \overline{c} \) \( (< 1) \). The post-innovation production marginal costs of the North firm and the South firm are denoted by \( c_n \) and \( c_s \) respectively. The two-way spillover parameter, \( \beta \), portrays involuntary flows of R&D output between the North and the South firms. The extent of spillovers can take two values: “no spillovers” or “complete spillovers”, \( \beta \in \{0,1\} \). This spillover parameter, \( \beta \) can also be interpreted as reflecting the strength of the IPR protection, implying no protection \( (\beta = 1) \) or full IPR protection \( (\beta = 0) \) and this is the interpretation we shall follow here. Let \( x_n \) and \( x_s \) denote the North and South firms’ autonomous R&D output respectively.

A firm’s R&D productivity is denoted by \( \theta_i \), and \( \theta_i \in (0,1) \). The two firms are asymmetric in the sense that firm \( s \) has a lower R&D productivity than firm \( n \): given the same effective R&D output\(^6\), \( s \) achieves less cost reduction than \( n \), or, equivalently, firm \( n \) is more productive than firm \( s \) in utilizing its R&D to bring down the unit cost of production, i.e., \( \theta_n > \theta_s \). This allows to take into account a certain degree of advantage \( n \) may have over \( s \) as, for example, a result of cumulative experience in conducting R&D. Without loss of generality, we normalize firm \( n \)'s R&D productivity to one, i.e. \( \theta_n = 1 \), and set \( \theta_s = \theta \) with \( \theta < 1 \).

Unit costs of production are therefore \( c_n = \overline{c} - (x_n + \beta x_s) \) and \( c_s = \overline{c} - \theta(x_s + \beta x_n) \). The R&D cost function takes the form: \( R_i = \frac{2x_i^2}{\gamma_i} \) where \( R_i \) denotes firm \( i \)'s R&D expenditure \( (i = n,s) \), and \( \gamma_i (> 0) \) captures R&D efficiency, which indicates how costly R&D is. When \( \gamma_i \) takes a high value, it means that a unit of R&D output can be achieved at high cost, thus the task of reducing unit cost is relatively difficult. The R&D cost function exhibiting diminishing returns to R&D. For simplicity, we assume that \( \gamma_n = \gamma_s = \gamma \). That is, to obtain a unit of R&D output, the firms spend equal sums of money.

We then consider a two-stage game where firms simultaneously and independently make decisions on R&D in the first stage, taking each others’ R&D decisions as given. They then compete in quantity in the second stage, given the level of R&D expenditure from the first stage of the game.

Our aim is to explore how different IPR protection regimes affect the North and the South firms’ R&D incentives, quantities and profits; in particular, we study no IPR protection vis-à-vis complete IPR protection. We thus divide our analysis into two cases: (i) no IPR protection (full spillovers, \( \beta = 1 \)) and (ii) complete IPR protection (no spillovers, \( \beta = 0 \)).

\(^5\)Firms’ R&D outputs are complementary so that a firm’s R&D knowledge can be useful to its rival as much as to itself.

\(^6\)The term “effective R&D output” is defined by Kamien et al. (1992) as the sum of a firm’s own R&D output and what it can extract from the other firms’ R&D outputs via spillovers.
2.1 No IPR Protection (NP or Full Spillovers, $\beta = 1$)

In this case, a firm can exploit the R&D output of the other firm as much as its own. The post-innovation unit cost of firm $n$ is $c_n = \tau - (x_n + x_s)$, and of firm $s$ is $c_s = \tau - \theta(x_n + x_s)$ and, obviously $c_s > c_n$.

In the second stage, each firm chooses its output to maximize profits, yielding two possible outcomes: an interior solution with

$$q_{NP}^i = \frac{1 - 2c_i + c_j x_i}{3}; i = n, s$$

requiring $c_s < \frac{c_n + 1}{2}$, and a corner solution, where $q_{NP}^s = 0$, $q_{NP}^n = \frac{1 - c_n}{2}$, occurring when $c_s \geq \frac{c_n + 1}{2}$. The cost combinations that generate the interior and corner solutions are shown in figure 1.

![Figure 1](image)

As we study how R&D decisions are affected by the IPR regime, we restrict the analysis to the case where both firms engage in R&D: $\theta > \frac{1}{2}$ is sufficient for such an interior solution$^7$.

**Assumption 1 (IS): For both firms to be active productivity is $\theta \in \left(\frac{1}{2}, 1\right)$.

Substituting for $c_n$ and $c_s$, (1) can be written as $q_{NP}^n = \frac{K + (2\theta - 1)(x_n + x_s)}{3}$ and $q_{NP}^s = \frac{K + (2\theta - 1)(x_n + x_s)}{3}$, where $K \equiv 1 - \tau > 0$ measures the "effective" market size; the associated equilibrium profits are $\pi_{NP}^n = (q_{NP}^n)^2$ and $\pi_{NP}^s = (q_{NP}^s)^2$. Observe that as a result of the complete spillovers between the North and the South, R&D benefits each other in terms of increasing quantity supplied ($\frac{\partial q_{NP}^n}{\partial x_s} = \frac{2 - \theta}{3} > 0$, and $\frac{\partial q_{NP}^s}{\partial x_n} = \frac{2 - \theta}{3} > 0$ (from Assumption IS)): $n$’s R&D output helps reduce $s$’s unit cost, leading to $n$’s output expansion and vice versa.

In the first stage of the game, each firm chooses R&D output to maximize second-stage profit net of R&D expenditure, i.e. $\pi_{NP}^n = \left(\frac{K + (2\theta - 1)(x_n + x_s)}{3}\right)^2 - \frac{\tau x_n^2}{2}$.

$^7$The requirement $c_s < \frac{c_n + 1}{2}$ can be written as $\tau - \theta(x_n + x_s) < \frac{\tau - (x_n + x_s) + 1}{2}$. By algebraic manipulation, it is easy to see that a sufficient condition for an interior solution ($x_n^* > 0$, and $x_s^* > 0$) is $\theta > \frac{1}{2}$. 

6
Table 1: Summary of the SPNE investments, outputs, profits and their comparative statics under weak IPRs protection regimes.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Equilibrium Value</th>
<th>sign ( \frac{\partial}{\partial \theta} )</th>
<th>Comparison Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{NP}^N )</td>
<td>( \frac{2K(2-\theta)(9\gamma+6(2\theta-1)(1-\theta))}{\Omega_f} )</td>
<td>(-)</td>
<td>( x_{NP}^N &gt; x_{NP}^S )</td>
</tr>
<tr>
<td>( x_{NP}^S )</td>
<td>( \frac{2K(2\theta-1)(9\gamma-6(2\theta-1)(1-\theta))}{\Omega_f} )</td>
<td>(-)</td>
<td>( x_{NP}^S &gt; x_{NP}^N )</td>
</tr>
<tr>
<td>( q_{NP}^N )</td>
<td>( \frac{9\gamma K(K+2(2\theta-1)(1-\theta))^2(9\gamma-2(2\theta-1)^2)}{\Omega_f} )</td>
<td>(-)</td>
<td>( q_{NP}^N &gt; q_{NP}^S )</td>
</tr>
<tr>
<td>( q_{NP}^S )</td>
<td>( \frac{9\gamma K(K-2(2\theta-1)(1-\theta))^2(9\gamma-2(2\theta-1)^2)}{\Omega_f} )</td>
<td>(-)</td>
<td>( q_{NP}^S &gt; q_{NP}^N )</td>
</tr>
</tbody>
</table>

and \( \pi_{NP}^N = \left( K+2(2\theta-1)(x_n+x_s) \right)^2 - \frac{\pi_2}{2} \). The corresponding first-order conditions give the firms’ best response functions:

\[ x_n = \frac{2(2-\theta)(K+(2-\theta)x_s)}{9\gamma-2(2-\theta)^2}, \]

(2)

\[ x_s = \frac{2(2\theta-1)(K+(2-1)x_n)}{9\gamma-2(2\theta-1)^2}. \]

(3)

The firms’ R&D, \( x_n \) and \( x_s \), are strategic complements\(^8\). Under complete spillovers, \( n \)'s R&D output helps reduce the unit cost of \( s \), thus enhances the marginal profitability of \( s \)'s R&D investment. A similar rationale applies to firm \( s \). From (2) and (3), solving for the equilibrium R&D outputs, \( x_{NP}^N \) and \( x_{NP}^S \), we obtain

\[ x_{NP}^N = \frac{2K(2-\theta)(9\gamma+6(2\theta-1)(1-\theta))}{\Omega_f}, \]

(4)

\[ x_{NP}^S = \frac{2K(2\theta-1)(9\gamma-6(2\theta-1)(1-\theta))}{\Omega_f}, \]

(5)

where \( \Omega_f \equiv [9\gamma-2(2-\theta)^2][9\gamma-2(2\theta-1)^2] - 4(2-\theta)^2(2\theta-1)^2 > 0 \) (from the relevant stability condition). Note that the additional assumption that \( \gamma > 1 \) is needed for the second-order conditions, the stability conditions and interior solutions to hold in the R&D subgame.\(^9\) The corresponding equilibrium values are summarized in Table 1, together with their comparative statics results.

The results in Table 1 show that the more R&D productive firm obtains a higher cost reduction (invests more in R&D), has a larger market share and profits more, as suggested by intuition. Interestingly, an increase in the productivity of the Southern firm is beneficial not only to itself but also to the rival North firm (\( \frac{\partial \pi_{NP}^N}{\partial \theta} > 0 \)) despite having a negative impact on the latter’s R&D and market share.

\(^8\) Derivation of these conditions and proofs of comparative static results are available from the authors upon request.

\(^9\) Derivation of these conditions and proofs of comparative static results are available from the authors upon request.
A rise in the South firm, s, R&D productivity increases its profitability and thus its incentive to invest. Even though both firms’ R&D are strategic complements, higher R&D by s will adversely affect the North firm’s, n, R&D. To understand this better, we disentangle the effects of R&D investment on n’s profit. From the first stage profit function \( \pi_n^{NP} = f(q_n^{NP}, q_s^{NP}, x_n) \), where \( q_n^{NP} \) and \( q_s^{NP} \) are the optimal quantities determined from the second stage:

\[
\frac{\partial \pi_n^{NP}}{\partial x_n} = \frac{\partial \pi_n^{NP}}{\partial q_n} \frac{\partial q_n^{NP}}{\partial x_n} \quad + \quad \frac{\partial \pi_n^{NP}}{\partial q_s} \frac{\partial q_s^{NP}}{\partial x_n} \quad + \quad \frac{\partial \pi_n^{NP}}{\partial x_n} \quad (6)
\]

We are interested in the strategic motive as it captures a firm’s intention to manipulate its rival’s R&D decision. Recall that \( \frac{\partial q_s^{NP}}{\partial x_n} > 0 \), so that the strategic motive is negative: n has an incentive to underinvest in R&D compared to the efficient level.\(^{10}\) This is because, n’s own R&D helps enhance the s’s quantity, which in turn reduces n’s profit. Once s’s R&D productivity increases, firm s invests more, thus supplying a higher quantity to the output market. Firm n foresees that its own R&D will just strengthen s’s position in the output market, it thus lowers its R&D. However, the increase in s’s R&D in response to the rise in its R&D productivity is not large enough to compensate for the fall in n’s investment and consequently n’s quantity falls. The interesting result is that n’s profit actually increases with s’s R&D productivity. This is mainly due to the savings n makes on its R&D expenditure: it freerides on s’s R&D and decreases its own.

### 2.2 Full IPR Protection (FP or No Spillovers \( \beta = 0 \))

When IPR protection is complete there are effectively no spillovers so a firm’s effective R&D is its own autonomous R&D output. Unit costs of the two firms are \( c_n = \pi - x_n \) and \( c_s = \pi - \theta x_s \). Equilibrium output in the second stage for each firm is still represented by (1). Substituting for \( c_n \) and \( c_s \), (1) can be written as \( q_n^{FP} = \frac{K + 2\theta x_n - \theta x_n}{\theta} \) and \( q_s^{FP} = \frac{K + 2\theta x_s - \theta x_n}{\theta} \). The associated equilibrium profit is \( \pi_i^{FP} = (q_i^{FP})^2, i = n, s \). Observe that \( \frac{\partial q_n^{FP}}{\partial x_n} = -\theta \), and \( \frac{\partial q_s^{FP}}{\partial x_s} = -\frac{1}{\theta} \), thus the effect of \( x_s \) on \( q_n^{FP} \) depends on s’s R&D productivity.

In the first stage of the game, each firm maximizes its own profit net R&D cost, taking R&D of the other firm as given. Thus the profit functions of the two firms are \( \pi_n^{FP} = (K + 2\theta x_n - \theta x_n)^2 - \gamma x_n^2, \pi_s^{FP} = (K + 2\theta x_s - \theta x_n)^2 - \gamma x_s^2 \).

The first-order conditions give the best response functions:

\[
\begin{align*}
x_n &= \frac{4K - 4\theta x_n}{\theta \gamma - 8}, \quad (8) \\
x_s &= \frac{4\theta K - 4\theta x_n}{\theta \gamma - 8\theta^2}. \quad (9)
\end{align*}
\]

\(^{10}\)The efficient level is determined by \( \frac{\partial \pi_n^{NP}}{\partial x_n} = \frac{\partial \pi_n^{NP}}{\partial q_n} \frac{\partial q_n^{NP}}{\partial x_n} + \frac{\partial \pi_n^{NP}}{\partial x_n} = 0 \); at this level of R&D, its marginal benefit is equated to its marginal cost. In this case, the efficient level is where \( q_n^{NP} - \gamma x_n = 0 \).
Table 2: Summary of the SPNE investments, outputs, profits and their comparative statics under complete IPRs protection regimes.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Values</th>
<th>Sign</th>
<th>Comparison Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_{FP} )</td>
<td>( \frac{4K(9\gamma - 12\theta^2)}{\Omega_n} ) &gt; 0</td>
<td>-</td>
<td>( x_{FP} &gt; x_{FP} )</td>
</tr>
<tr>
<td>( x_{FP} )</td>
<td>( \frac{4\theta K(9\gamma - 12)}{\Omega_n} ) &gt; 0</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>( q_{FP} )</td>
<td>( \frac{9\gamma K(9\gamma - 4\theta^2)}{\Omega_n} ) &gt; 0</td>
<td>-</td>
<td>( q_{FP} &gt; q_{FP} )</td>
</tr>
<tr>
<td>( q_{FP} )</td>
<td>( \frac{9\gamma K(9\gamma - 4\theta^2)}{\Omega_n} ) &gt; 0</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>( \pi_{FP} )</td>
<td>( \frac{9\gamma K(9\gamma - 4\theta^2)(9\gamma - 8)^2}{4\Omega_n^2} &gt; 0 )</td>
<td>-</td>
<td>( \pi_{FP} &gt; \pi_{FP} )</td>
</tr>
<tr>
<td>( \pi_{FP} )</td>
<td>( \frac{9\gamma K(9\gamma - 4\theta^2)(9\gamma - 8)^2}{4\Omega_n^2} &gt; 0 )</td>
<td>+</td>
<td></td>
</tr>
</tbody>
</table>

The two firms’ R&D are strategic substitutes\(^{11}\). Under full IPR protection, R&D for the two firms is a strategic substitute. An expansion in the North’s output supplied induced by its own R&D investment adversely affects the South’s profit. That means an increase in \( n \)’s R&D reduces the marginal profitability of a unit of R&D output achieved by \( s \). Consequently \( s \) cuts down its own R&D.

From (8) and (9) solving for equilibrium R&D we obtain

\[
\begin{align*}
x_{n_{FP}} &= \frac{4K(9\gamma - 12\theta^2)}{\Omega_n}, \\
x_{s_{FP}} &= \frac{4\theta K(9\gamma - 12)}{\Omega_n},
\end{align*}
\]

where \( \Omega_n \equiv (9\gamma - 8)(9\gamma - 8\theta^2) - 16\theta^2 > 0 \) from the relevant stability condition. For the second-order condition, stability condition and positive R&D investments to hold in this R&D subgame, we need \( \gamma > \frac{12}{4\theta} \).\(^{12}\)

The corresponding equilibrium values and how they respond to the change in \( s \)’s R&D productivity are summarized in Table 2.

Similar to the previous case of no IPR protection, the more R&D productive firm attains higher R&D marginal profitability, thus has higher incentive to invest, which consequently leads to a larger reduction in production cost, more quantity supplied to the market, and higher profit made.

Higher R&D by the \( s \) due to the increased R&D productivity adversely affects \( n \)’s R&D, quantity and profit. To see the rationale behind, we again disentangle the effects of R&D investment on \( n \)’s profit. From the first stage profit function: \( \pi_{FP} = P(\hat{\eta}_n, q_{FP})\hat{\eta}_n - (\pi - x_n)\hat{\eta}_n - \frac{\gamma^2}{2} \), where \( q_{FP} \) and \( q_{FP} \) are the optimal quantities determined from the second stage, we disentangle the effect of \( x_n \) on \( \pi_{FP} \)

\[
\frac{d \pi_{FP}}{dx_n} = \frac{\partial \pi_{FP}}{\partial q_n} \frac{dq_n}{dx_n} + \frac{\partial \pi_{FP}}{\partial q_s} \frac{dq_s}{dx_n} + \frac{\partial \pi_{FP}}{\partial x_n}
\]

0 from FOC in 2nd stage strategic motive profit motive

\[
= 0 + \hat{\eta}_n \frac{dq_n}{dx_n} + \frac{\partial \pi_{FP}}{\partial x_n}
\]

\(^{11}\) Derivation of these condition and proofs of comparative statics results are available from the authors upon request.
Recall that $\frac{\partial q_{FP}}{\partial x_n} < 0$ when spillovers are absent. The strategic motive is positive which means $n$ has an incentive to overinvest in R&D compared to the efficient level.\textsuperscript{13} $n$’s R&D negatively affects $s$’s quantity, which consequently increases $n$’s profit. Thus, $n$ has incentive to overinvest. An increase in $s$’s R&D productivity induces higher investment and larger quantity supplied by $s$ in the output market. $n$ knows that its own R&D will not be as effective in reducing $s$’s quantity in the output market. Thus, its incentive to overinvest declines. Since $n$ cannot benefit from any increase in R&D conducted by $s$ under strict IPR protection, its quantity supplied falls as a result of lower R&D. Consequently, $n$’s profit fails.

2.3 Comparison of IPR Regimes

In this section we compare IPR regimes. From the analysis above we have established that the North firm, $n$, does more R&D, produces more output and profits more than the South firm, $s$, irrespective of the IPR regime, due to the R&D productivity difference.

Effective R&D under no IPR protection, i.e. $x_{NP}^n + x_{NP}^s$, is $\frac{18\gamma K(1+\theta)}{17}$. It is straightforward to show\textsuperscript{14} that within the admissible values for the productivity parameter (i.e., $\theta \in (1/2, 1]$) and as long as $\gamma > 1.5$ effective R&D is increasing in R&D productivity, i.e., $\frac{d(x_{NP}^n + x_{NP}^s)}{dn} > 0$. In what follows, this latter restriction on $\gamma$ is imposed. Comparing $x_{NP}^n + x_{NP}^s$ with $x_{FP}^n$ and $x_{FP}^s$, we find that $x_{FP}^s > x_{NP}^s + x_{NP}^n$ for $\theta \in (1/2, 1)$ and $x_{FP}^n = x_{NP}^n + x_{NP}^s$ for $\theta = 1$. So unless R&D productivity is the same for both firms, the effective R&D of $n$ is higher under full IPR protection compared to that under no IPR protection. Next, comparing $x_{NP}^n + x_{NP}^s$ and $x_{FP}^n$, we find that unless $\theta = 1$, $s$’s R&D under complete IPR protection is less than effective R&D under no IPR protection (i.e., $x_{FP}^s < x_{NP}^s + x_{NP}^n$). At $\theta = 1$, we have $x_{NP}^n + x_{NP}^s = x_{FP}^n$. So to summarise, $s$’s autonomous R&D under full IPR protection is less than its effective R&D under no IPR protection except when $\theta = 1$, where effective R&D levels under the two contrasting IPR regimes are equal.

Regarding profits the following proposition summarises (its proof is in the Appendix).

**Proposition 1** (i) The South firm always profits more under no IPR protection than under full IPR protection, $\pi_{NP}^s > \pi_{FP}^s$.

(ii) As per the North firm, there exists a critical value $\bar{\theta}$ such that $\pi_{NP}^n > \pi_{FP}^n$ for $\theta > \bar{\theta}$, and $\pi_{NP}^n \leq \pi_{FP}^n$ otherwise.

Interestingly, there are cases where $n$ prefers no IPR protection: this is so when $s$’s R&D productivity is high enough ($\theta > \bar{\theta}$). In these instances $n$ profits more from no IPR protection compared to the full IPR protection regime. The range where this is the case increases as R&D becomes more difficult, i.e.,

\textsuperscript{13} The efficient level is determined by $\frac{\partial q_n}{\partial q_n}d_{\theta n} + \frac{\partial q_n}{\partial x_n}$. At this level of R&D, its marginal benefit is equated to its marginal cost.

\textsuperscript{14} Proofs of $\frac{d(x_{NP}^n + x_{NP}^s)}{dn} > 0$, the comparison between $x_{NP}^n + x_{NP}^s$ and $x_{FP}^n$, $x_{FP}^s$ and further comparisons are available from the authors upon request.
the critical value $\tilde{\theta}$ decreases in $\gamma$, the difficulty/costliness of R&D. This is illustrated in Figure 2 and Table 3.  

![Figure 2](image)

Table 3 - Critical value $\tilde{\theta}$

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>1.5</th>
<th>1.78</th>
<th>2</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{\theta}$</td>
<td>0.89</td>
<td>0.80</td>
<td>0.77</td>
<td>0.65</td>
<td>0.63</td>
<td>0.62</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Regarding the North firm’s interest in IPR protection, our model produces results different from Chin and Grossman (1990). In their model of a single innovator, in which only one firm in the North performs R&D activities, the North firm always prefers high IPR protection as its profit is higher when the South firm cannot benefit from any copying. In contrast, in our model, the North firm has an opportunity to make use of the South firm’s technology for its own benefit. In particular, if the South firm is relatively productive in R&D, its incentive perform R&D is of benefit to the North firm despite IPR not being fully protected. In summary, although the South firm always prefers no IPR to complete IPR as expected, there are cases where the North firm has the same preference as the South firm for the IPR regime so that the traditional conflict about IPR regimes does not hold. This novel results depends on the magnitude of the South firm’s R&D productivity, the higher the R&D productivity the less the conflict about the extent of IPR protection.

3 Lobbying for IPR protection

When there is conflict in the preference towards IPR regime by the rival firms an interesting question to ask is whether if the firms were given the option of side payments between them, would it be in their interest to lobby for a particular type of IPR protection. In such a case, side payments may be used to alleviate the firms’ conflict of interest.

From proposition 1 we know that $\pi_{n}^{NP} > \pi_{n}^{FP}$ for $1 > \tilde{\theta} > \theta$ and $\pi_{n}^{NP} \leq \pi_{n}^{FP}$ for $\frac{1}{2} < \theta < \tilde{\theta}$, whereas $\pi_{s}^{NP} > \pi_{s}^{FP}$ for all $\frac{1}{2} < \theta < 1$. The conflict of interest arises when $\frac{1}{2} < \theta < \tilde{\theta}$. In this range of $\theta$, if the North firm finds $\pi_{n}^{FP} - \pi_{n}^{NP} > \pi_{s}^{NP} - \pi_{s}^{FP}$ (or, $\pi_{n}^{FP} + \pi_{s}^{FP} > \pi_{n}^{NP} + \pi_{s}^{NP}$) it may pay $\pi_{s}^{NP} - \pi_{s}^{FP}$ to the South.

\footnote{Since the expression for $\tilde{\theta}(\gamma)$ is quite cumbersome we calculate the value of $\tilde{\theta}(\gamma)$ for fixed $K$, as this is just a scaling parameter. In Table 3, we have fixed $K = 1$.}

\footnote{In Figure 2, we choose $\gamma = 2$ and $\gamma = 10$ as examples. These values of $\gamma$ are sufficient for the stability condition to hold in both regimes and $K$ is fixed at 1.}
firm on the condition that the South firm lobbies its government to adopt the strict IPR regime. Similarly, if the South firm finds \( \pi_{s}^{NP} - \pi_{s}^{FP} > \pi_{n}^{FP} - \pi_{n}^{NP} \) or \( \pi_{s}^{FP} + \pi_{n}^{FP} < \pi_{n}^{NP} + \pi_{s}^{NP} \) it can pay the North firm \( \pi_{n}^{FP} - \pi_{n}^{NP} \) and ask the North to lobby its government to go for lax IPR protection.

We then search for a range of \( \theta \) that gives either \( \pi_{n}^{FP} - \pi_{n}^{NP} > \pi_{s}^{NP} - \pi_{s}^{FP} \) or \( \pi_{s}^{NP} - \pi_{s}^{FP} > \pi_{n}^{FP} - \pi_{n}^{NP} \). From the equilibrium expressions for profit we calculate industry profit under the two regimes:

\[
\Pi^{NP} = \pi_{n}^{NP} + \pi_{s}^{NP} = \frac{18K^{2}\gamma(81\gamma^{3} - 9\gamma^{2}(23 + \theta(23\theta - 44)))}{162\gamma(1 - \theta)^{3} - 8(1 - 2\theta)^{2}(2 - \theta)^{2}(1 - \theta)^{2}} \Omega_{f}^{2}
\]

\[
\Pi^{FP} = \pi_{n}^{FP} + \pi_{s}^{FP} = \frac{18K^{2}\gamma(81\gamma^{3} - 144\gamma^{2}(1 + \theta^{2})) + 24\gamma(3 + 8\theta^{2} + 3\theta^{4}) - 64\theta^{2}(1 + \theta)^{2}}{\Omega_{n}^{2}}
\]

Due to the complexity of the above expressions, we resorted to numerical simulations. Figure 3 shows the relationship between the South firms’s R&D productivity, R&D effectiveness and industry profits under no IPR protection and full protection, \( \Pi^{NP} \) and \( \Pi^{FP} \). In the Figure, shown for \( K = 10 \) (this is a scaling parameter that does not affect the shapes of the profit surfaces), \( \Pi^{NP} \) is shown in red (top surface) and \( \Pi^{FP} \) is shown in blue (bottom surface).

\[
\text{Figure 3}
\]

It is clear that \( \Pi^{NP} > \Pi^{FP} \), or \( \pi_{n}^{NP} + \pi_{s}^{NP} > \pi_{n}^{FP} + \pi_{s}^{FP} \), is always the case for \( \frac{1}{2} < \theta < 1 \). Industry profit under no IPR protection is larger than that under strict IPR protection. Therefore the North firm will find the offer from the South firm in exchange for the adoption of no IPR protection regime beneficial. We then conclude that if \( \frac{1}{2} < \theta < \hat{\theta} \), the conflict of interest between the North and the South firms can be alleviated using a side payment scheme.
4 Voluntary Information Exchange

In this section we confine our analysis within the strict IPR protection regime (e.g., enforced by the governments of both countries) and investigate whether the firms would find it in their interest to voluntarily engage in information exchange. From section 2, we know that the South firm would always prefer a lax IPR regime while the North firm would only do so for relatively high R&D productivity of the South firm. When the strict IPR regime is enforced, engaging in information exchange might serve as a way-round the IPR regime. Thus we extend the basic model to a three-stage game: in the first stage firms make decisions simultaneously and noncooperatively on their R&D; in the second-stage they decide whether to share or not their first-stage discoveries (information exchange stage) and then compete in the product market in the third stage. Only when both firms agree to share information does exchange of information occur (Kultti and Takalo (1998)).

We start by having \( x_n^v \) and \( x_s^v \) denote the levels of (sunk) R&D made in the first stage while \( n \) and \( s \) are the degrees of information exchange realized by \( n \) and \( s \) respectively. We allow for differences in R&D productivity of the two firms; \( \theta_n \) and \( \theta_s \) are the R&D productivity parameters respectively and \( \theta_n > \theta_s \).

In the second stage, exchange of information happens only when such exchange benefits both firms in terms of higher profits. Thus in deciding, firms compare profits obtained under no agreement with those when there is agreement.

Solving for firm’s \( n \) profit under a given degree of information sharing we find

\[
\pi_{n}^{IE} = \left( 1 - \pi + \frac{2\theta_n - \theta_s \beta_n}{3} x_n^v + \frac{(2\theta_n \beta_n - \theta_s) x_s^v}{2} \right)^2 - \frac{\gamma (x_n^v)^2}{2}. \tag{14}
\]

In the case of no information exchange

\[
\pi_{n}^{NIE} = \left( 1 - \pi + \frac{2\theta_n x_n^v - \theta_s x_s^v}{2} \right)^2 - \frac{\gamma (x_n^v)^2}{2}. \tag{15}
\]

Obviously, \( \pi_{n}^{IE} > \pi_{n}^{NIE} \) if and only if \( \theta_n \beta_n x_n^v > \theta_s x_s^v \). This means that \( n \) welcomes the agreement only if the "net" benefits (i.e., adjusted for productivity differences) from R&D spillovers from \( s \) is larger than one half of what \( n \) contributes the \( s \)'s cost reduction.

Similarly for firm \( s \) we obtain,

\[
\pi_{s}^{IE} = \left( 1 - \pi + \frac{2\theta_s \beta_s - \theta_n}{3} x_n^v + \frac{(2\theta_s - \theta_n \beta_s) x_s^v}{2} \right)^2 - \frac{\gamma (x_s^v)^2}{2}, \tag{16}
\]

and

\[
\pi_{s}^{NIE} = \left( 1 - \pi - \theta_n x_n^v + \frac{2\theta_s x_s^v}{2} \right)^2 - \frac{\gamma (x_s^v)^2}{2}. \tag{17}
\]

Clearly \( \pi_{s}^{IE} > \pi_{s}^{NIE} \) if and only if \( \theta_s x_s^v > \frac{\theta_n \beta_s x_n^v}{2} \) with a similar interpretation as for \( n \) above.
For simplicity, suppose that $\beta_s = \beta_n = \beta$. The firms mutually agree on a particular level of sharing, which implies that the exchange of information will occur only when $\frac{\theta_n x_n^v}{2} < \theta_n x_n^v < 2\theta_n x_n^v$ or $\frac{1}{2} < \frac{\theta_s x_s^v}{\theta_s x_n^v} < 2$. In deciding whether to share information or not in the second stage, a firm takes into account the levels of sunk R&D of its counterpart and its R&D productivity. The relative benefit of the agreement accrued to $n$ and $s$, $\frac{\theta_s x_s^v}{\theta_n x_n^v}$ must be in the range $(\frac{1}{2}, 2)$.\textsuperscript{17}

In the first stage, the firms independently and simultaneously make decisions on R&D taking into account the possibility of an information exchange agreement in the second stage. Differentiating (14) and (16) with respect to $x_s^v$ and $x_n^v$ respectively, we obtain the firms’ best responses: $x_n^v = \frac{2(2\theta_n - \theta_s)(K + (2\theta_n - \theta_s)x_n^v)}{9\gamma - 2(2\theta_n - \theta_s)x_n^v}$ and $x_s^v = \frac{2(2\theta_s - \theta_n)(K + (2\theta_s - \theta_n)x_s^v)}{9\gamma - 2(2\theta_s - \theta_n)x_s^v}$. Solving\textsuperscript{18} for the SPE we obtain

$$x_n^v = \frac{2(2\theta_n - \theta_s \beta)K[9\gamma - 6(2\theta_n - \theta_s \beta)(\theta_s - \theta_n \beta)]}{\Phi},$$

$$x_s^v = \frac{2(2\theta_s - \theta_n \beta)K[9\gamma - 6(2\theta_n - \theta_s \beta)(\theta_n - \theta_s \beta)]}{\Phi},$$

where $\Phi = [9\gamma - 2(2\theta_n - \theta_s \beta)^2][9\gamma - 2(2\theta_n - \theta_s \beta)^2] - 4(2\theta_s - \theta_n \beta)(2\theta_n - \theta_s \beta)(2\theta_n - \theta_s \beta)(2\theta_s - \theta_n \beta)(2\theta_n - \theta_s \beta) > 0$. An interior solution is guaranteed when $\theta_i > \frac{\theta_i}{2}$, $i = n, s$.

Since the condition $\frac{\theta_n x_n^v}{2} < \theta_s x_s^v < 2\theta_n x_n^v$ must hold for voluntary exchange of information, we examine whether $2\theta_n x_n^v - \theta_s x_s^v > 0$ and $\theta_s x_s^v - \frac{\theta_s x_s^v}{2} > 0$ hold. From (18)

$$2\theta_n x_n^v - \theta_s x_s^v = \frac{2K[9\gamma(2\theta_n \theta_s - 2\beta \theta_n^2 + \beta \theta_n^2)] - 6(2\theta_n - \beta \theta_n)(2\theta_n - \theta_s \beta)(2\theta_n - \theta_s \beta - \theta_n^2)]}{\Phi},$$

$$\theta_s x_s^v - \frac{\theta_s x_s^v}{2} = \frac{K[9\gamma(2\theta_n \theta_s - 2\beta \theta_n^2 + \beta \theta_n^2)] - 6(2\theta_n - \beta \theta_n)(2\theta_n - \theta_s \beta)(2\theta_n - \theta_s \beta - \theta_n^2)]}{\Phi}.$$ \textsuperscript{(20a) (20b)}

Due to the complexity of the above two expressions, we use numerical simulations to facilitate the comparison. We fix $\theta_n$ at 1, and first set $\gamma$ at the minimum value compatible with the second-order and stability conditions being met, i.e., $\gamma = \frac{16}{19}$. We then observe how (20a) and (20b) vary with $\theta_n$ and $\beta$. In Figure 4 below, panels (a) and (b) illustrate the contour plot (20a) and (20b) of respectively. The white region indicates irrelevant combinations of ($\theta_n$, $\beta$), i.e., those that do not lead to an interior solution ($\theta_n \leq \frac{\beta}{2}$). The dark grey region in panel (a) shows the combination of $\theta_n$ and $\beta$ that give $2\theta_n x_n^v - \theta_s x_s^v < 0$, while the light grey region indicates the opposite. In panel (b) the light grey region shows that $\theta_n x_n^v - \frac{\theta_s x_s^v}{2} > 0$ for all admissible values of $\theta_n$ and $\beta$. It is then the case that both $2\theta_n x_n^v - \theta_s x_s^v > 0$ and $\theta_s x_s^v - \frac{\theta_s x_s^v}{2} > 0$ hold in the light grey

\textsuperscript{17}In the special case of $\theta = 1$, i.e., firms are symmetric, any degree of voluntary information sharing is always optimal as $\frac{\theta_s x_s^v}{\theta_n x_n^v} < x_n^v < 2x_n^v$ holds for all $x_n^v = x_s^v = x^v$, as obtained by Kultti and Takalo (1998).

\textsuperscript{18}To satisfy the relevant second order and stability conditions it is necessary to impose the restriction that $\gamma > \frac{10}{9}$. 

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region of panel (a); this is the area showing combinations of $\theta_s$ and $\beta$ that result in information exchange. The following Claim summarises this discussion.

**Claim 2** Given $\gamma = 16.1/9$, $\theta_n = 1$, $\theta_s > \frac{\theta}{\gamma}$, the South firm always finds information exchange beneficial for all possible values of $\beta$ and $\theta_s$. On the contrary, only for $\theta_s$ sufficient high does the North firm find information exchange beneficial for all possible values of $\beta$.

Next, we explore how the region where both firms engage in information exchange changes as R&D becomes more difficult, i.e., we change the values of the parameter $\gamma$. Figure 5 illustrates for selected values of $\gamma$, ranging from very easy R&D (low value) to difficult R&D (high value). The dark gray area diminishes as $\gamma$ increases: as R&D becomes more difficult/costly, the area where both firms engage in information exchange increases, this is so for even low $\theta_s$. As the North firm faces an inferior South firm competitor, it compares the gain from sharing information with the gain from forgoing the sharing agreement, when making its R&D decision. When R&D is not relatively costly, and the South firm is not very productive in R&D, the North finds that it does better not sharing information in a wide range of cases (combinations of $\theta_s$ and $\beta$); as R&D becomes more costly the gain from exchanging information even with a low R&D productivity rival outweighs the loss of not pursuing information exchange and sharing information is generally better for both firms.
In summary then, even when IPR are strictly protected, the R&D investments made by the North and the South firms are sufficient to induce information exchange.

5 Conclusion

In this paper, we analyze the impact of different regimes of IPR protection on the performance of asymmetric duopolists in a North-South context where the South firm has a capability to perform process R&D but at a lower productivity compared to the North firm. Our analysis allows to examine how firms’ investment incentives and their profits differ across two contrasting IPR protection regimes, and also how the firms’ R&D decisions are affected by the level of the South firm’s R&D productivity. These results could shed some light on the governments’ policy on the pursuit or not of enforcing strict IPR protection regimes.
The major finding that gives our model novelty is that the North firm earns higher profit when the South country does not protect the North firm’s IPR compared to what it would obtain had the IPR protection been strictly enforced, provided that the South firm’s R&D productivity is not too low. Also, the South firm prefers strict IPR protection regime if its R&D productivity is too low to make adequate use of spillovers from the North under weak IPR protection. By allowing for the possibility of process innovation by the firm in the South, our results contrast with those derived by Chin and Grossman (1990), Diwan and Rodrik (1991), Deardorff (1992) and Zigić (1998), where a conflict of interests with respect to IPRs between North and South is the norm. In our model, a consensus on the IPR protection regime can be reached when the South firm’s R&D productivity is relatively high but the conflict between North and South seems to be inevitable if the South firm’s R&D productivity is not high enough.

We also find that in the case where conflicts are inevitable, the option of side payments can be used to induce firms to agree upon a particular regime of IPR protection. Unless the R&D is very cheap to deliver industry profit is higher when IPR are not protected. The benefit from making savings on the firm’s own R&D expense via free-riding on other firm’s R&D is significant. The North firm finds side payment offered by the South in exchange for the North’s adoption of no IPR protection regime beneficial.

In addition, we investigate the case where IPR protection is strictly enforced and firms are allowed to voluntarily engage in information exchange. With a uniform degree of sharing set for both sides, any degree of information sharing is beneficial as long as the North firm can get in terms of knowledge spillovers from the South firm is greater than a half of what the South firm can benefit from the North firm’s R&D and what the South firm obtains in terms of productive R&D is larger than a half of what it gives to the North firm, even for low values of the South firm’s R&D productivity.

6 Appendix

6.1 Proof of Proposition 1

Since both $\pi_{s}^{FP}$, $\pi_{s}^{NP}$ increase with $\theta$, they attain their minima at $\theta = 1/2$, and reach their maxima at $\theta = 1$. We show that the minimum of $\pi_{s}^{NP}$ is higher than the maximum of $\pi_{s}^{FP}$, by evaluating $\pi_{s}^{NP}$ at $\theta = 1$ (for its maximum) and $\pi_{s}^{NP}$ at $\theta = 1/2$ (for its minimum). From $\pi_{s}^{NP}|_{\theta=1/2} = K^{2}$ and $\pi_{s}^{NP}|_{\theta=1} = \frac{\gamma K^{2}(9\gamma-8)}{(9\gamma-4)^{2}}$, we have $\pi_{s}^{NP}|_{\theta=1/2} - \pi_{s}^{NP}|_{\theta=1} = \frac{16K^{2}}{9(9\gamma-4)^{2}} > 0$. Thus for $\gamma > \frac{12}{9}$ and $\theta \in (\frac{1}{2}, 1)$, $\pi_{s}^{NP} < \pi_{s}^{NP}$. From $\frac{d\pi_{s}^{FP}}{d\theta} < 0$ and $\frac{d\pi_{s}^{NP}}{d\theta} > 0$ for $\theta \in (\frac{1}{2}, 1)$, it is implied that $\pi_{n}^{NP}$ reaches its maximum at $\theta = 1$ and its minimum at $\theta = \frac{1}{2}$, whereas $\pi_{n}^{FP}$ reaches its maximum at $\theta = \frac{1}{2}$ and its minimum at $\theta = 1$. Next, we examine if there is any $\theta$ such that $\pi_{n}^{FP} < \pi_{n}^{NP}$; in doing so we evaluate $\pi_{n}^{NP}$ and $\pi_{n}^{FP}$ at their maxima and minima.

At $\theta = \frac{1}{2}$, $\pi_{n}^{FP}|_{\theta=1/2} = \frac{2\gamma K^{2}(3\gamma-1)^{2}(9\gamma-8)}{(9\gamma-4)^{2}(9\gamma-2-4)^{2}}$ and $\pi_{n}^{NP}|_{\theta=1/2} = \frac{2\gamma K^{2}}{9(2\gamma-1)^{2}}$, so $\pi_{n}^{FP}|_{\theta=1/2} - \pi_{n}^{NP}|_{\theta=1/2} = \frac{2\gamma K^{2}(40+3(3(2\gamma-8)-59))}{9(9\gamma-8)(9\gamma-2-4)(2\gamma-1)^{2}} > 0$. However, as $\gamma > \frac{12}{9}$ (from the stability condition), the denominator of this expression is positive. The term $3\gamma(27\gamma-8)-59$ reaches its minimum of 2.53 at $\gamma = \frac{12}{9}$. Therefore
At \( \theta = 1 \), \( \pi_{n}^{FP} \mid \theta = 1 \) = \( \frac{\gamma K^2(\gamma-8)}{(9\gamma-4)^2} \) and \( \pi_{n}^{NP} \mid \theta = 1 \) = \( \frac{\gamma K^2(\gamma-2)}{(9\gamma-4)^2} \), so \( \pi_{n}^{FP} \mid \theta = 1 - \pi_{n}^{NP} \mid \theta = 1 = \frac{-6\gamma K^2(\gamma-4)}{(9\gamma-4)^2} < 0 \), therefore \( \pi_{n}^{FP} \mid \theta = 1 < \pi_{n}^{NP} \mid \theta = 1 \).

Thus, with \( \frac{d\pi_{n}^{FP}}{d\theta} < 0 \) and \( \frac{d\pi_{n}^{NP}}{d\theta} > 0 \) the sign of \( (\pi_{n}^{FP} - \pi_{n}^{NP}) \) changes from positive to negative as we move from \( \theta = 1/2 \) to \( \theta = 1 \). This implies that \( \pi_{n}^{FP} \) and \( \pi_{n}^{NP} \) intersect once within \( \theta \in \left(\frac{1}{2}, 1\right] \). In other words, for \( \gamma > \frac{12}{9} \) and \( \theta \in \left(\frac{1}{2}, 1\right] \), there exists a critical value of \( \theta \), \( \hat{\theta} \) such that for \( \theta > \hat{\theta} \), \( \pi_{n}^{FP} < \pi_{n}^{NP} \), and \( \pi_{n}^{FP} > \pi_{n}^{NP} \) otherwise.
References


