Patent protection under endogenous product differentiation.

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Abstract: It is generally believed that a weak patent protection makes the consumers and the society better off compared to a strong patent protection by increasing the intensity of competition if the weak patent protection does not affect innovation. We show that this conclusion may not hold if the innovator can take other non-production strategies, such as product differentiation, to reduce the intensity of product-market competition. A weak patent protection may reduce consumer surplus and social welfare by inducing product differentiation by the innovator. We show that the type of product-market competition and the market demand function play important roles in this respect. Hence, there can be an argument for a strong patent protection even if it does not affect innovation.

Key Words: Patent protection; Product differentiation; Welfare

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1. Introduction

An important role of the patent system is to increase welfare by enhancing innovation. The basic argument goes as follows. If innovation occurs and there is no (or weak) patent protection, imitation or knowledge spillover allows more firms than the original innovator to use the innovated technology,\(^1\) thus increasing competition in the product market. However, a weak patent protection may not encourage the innovator to invent the new technology, thus creating a negative effect on the consumers and the society.\(^2\) Hence, if a weak patent protection does not affect the incentive for innovation significantly, the first effect generally dominates the second effect and a weak patent protection makes the consumers better off and generally makes the society better off compared to a strong patent protection.\(^3\)

Although the above argument is certainly intuitive and appealing, the literature on patent protection is restrictive by focusing mainly on innovation\(^4\) and ignoring other non-production activities of the innovator, such as investment in product differentiation. We take up this issue here and show that even if a weak patent protection does not affect innovation, it may make the consumers and the society

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\(^1\) Mansfield et al. (1981) find that 60% of a sample of their patented innovations is imitated within four years and the average cost of imitation is two-thirds the original cost of innovation.

\(^2\) See, Mazzoleni and Nelson (1998) provide an overview on the benefits and costs of patent protection.

\(^3\) One can argue following the result of Lahiri and Ono (1988) that a weak patent protection reduces welfare even if it does not affect innovation but it reduces the marginal cost of a high-cost firm slightly and the marginal cost difference between the firms is sufficiently large. However, a weak patent protection makes the consumers better off in Lahiri and Ono (1988).

worse off compared to a strong patent protection by inducing the innovator to reduce competition through product differentiation.\(^5\)

We consider a simple model with an innovating firm and a non-innovating firm to show how a weak patent protection, which reduces the non-innovating firm’s marginal cost of production, affects the innovating firm’s incentive for product differentiation, consumer surplus and social welfare. We do our analysis under Cournot and Bertrand competition and for the demand functions due to Shubik and Levitan (1980), where product differentiation does not affect the market size, and due to Bowley (1924), where higher product differentiation increases the market size.

We consider a Shubik and Levitan type demand function in Section 2. Hence, the role of product differentiation in that section is only to reduce product-market competition. We show that a weak patent protection increases the innovator’s incentive for product differentiation and may make the consumers and the society worse off compared to a strong patent protection under both Cournot and Bertrand competition.

Section 3 consider a Bowley type demand function, where product differentiation reduces competition in the product market but helps to increase the total market size. The market expansion effect of product differentiation tends to reduce the negative competition reducing effect of product differentiation on the consumers and the society. We show that even under a Bowley type demand function, a weak patent protection increases the innovator’s incentive for product differentiation. However, whether a weak patent protection makes the consumers and the society worse off by inducing product differentiation by the innovator may depend

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on the type of product-market competition. While a weak patent protection makes the consumers and the society better off under Cournot competition, it may make the consumers and the society worse off under Bertrand competition.

To eliminate the effects of patent protection on the innovator’s incentive for process innovation, we assume away any cost of process innovation. We also assume a large marginal cost reduction under weak patent protection to eliminate the reason for a strong patent protection following Lahiri and Ono (1988), which shows that if the marginal cost difference between the firms is large, a small cost reduction in the high-cost firm reduces welfare, thus providing a reason for a strong patent protection. Thus, our paper provides a new reason for a strong patent protection even if it does not affect innovation.

The remainder of the paper is organised as follows. Section 2 describes the basic model. Section 3 considers a Shubik and Levitan type demand function and shows the results under Cournot and Bertrand competition. Section 4 considers a Bowley type demand function and shows the results under Cournot and Bertrand competition. Section 5 concludes.

2. The basic model

Assume that there are two firms, 1 and 2, producing $x$ and $y$. Each firm can produce the respective product at a constant marginal cost $c$. Firm 1 innovates a process technology that reduces its marginal cost to $c_1$, which is assumed to be 0, for simplicity. If the patent protection in the economy is strong, we assume that innovation by firm 1 does not affect firm 2’s marginal cost. In this situation, the marginal cost of firm 2 remains at $c$. However, if the patent protection in the economy is weak, we assume that innovation by firm 1 creates knowledge spillover (or
imitation) and allows firm 2 to produce at a constant marginal that is lower than \( c \). We will assume in our analysis that the marginal cost of firm 2 is 0 under weak patent protection. This is to avoid the reason for a strong patent protection following Lahiri and Ono (1988).

To eliminate the effect of the patent system on firm 1’s incentive for process innovation, we assume that firm 1 innovates irrespective of the patent system. This happens if there is no or sufficiently small cost associated with process innovation. We assume for simplicity that there is no cost of process innovation.

We will consider two types of demand functions: (i) due to Shubik and Levitan (1980), and (ii) due to Bowley (1924). As shown below, the demand function due to Shubik and Levitan (1980) shows that the market size is independent of the degree of product differentiation, while the demand function due to Bowley (1924) shows that the market size increases with higher product differentiation. To capture both types of demand functions, we assume that a representative consumer's utility function is

\[
U = (x + y) - [1 + s(1 - g)] \frac{1}{2} (x^2 + y^2) - gxy + \xi ,
\]

where \( \xi \) is the numeraire good and \( g \in [0,1] \) shows the degree of product differentiation. If \( g=0 \), the goods are isolated and if \( g=1 \), the goods are perfect substitutes. The parameter \( s \in [0,1] \) measures the degree of market expansion, where \( s=1 \) corresponds to no market expansion effect, as in Shubik and Levitan (1980), and \( s=0 \) generates a preference function due to Bowley (1924), which shows that the market size increases with a higher product differentiation.

The inverse market demand functions for \( x \) and \( y \) generated from (1) are:

\[
P_x = 1 - \alpha x - gy
\]
\[ P_y = 1 - \alpha y - gx \]  
\[ \alpha = [1 + s(1 - g)] \]  

To provide a better understanding of the market expansion effect, we aggregate the demand functions, which take the form of  
\[ (x + y) = [1 + g + s(1 - g)]^{-1} 2(1 - \bar{P}) \], where \( \bar{P} = \frac{P_1 + P_2}{2} \) is the average price. If \( s = 1 \), we get \( (x + y) = (1 - \bar{P}) \), suggesting that the total demand is independent of \( g \), as in Shubik and Levitan (1980). However, if \( s = 0 \), we get \( (x + y) = [1 + g]^{-1} 2(1 - \bar{P}) \), suggesting that a lower \( g \) increases the total demand, i.e., the market size increases with a lower \( g \), as in Bowley (1924).

To start with, we assume that \( x \) and \( y \) are perfect substitutes. However, firm 1 can invest certain amount to differentiate its product from that of firm 2. We assume that firm 1 can create \( g \) degree of horizontal product differentiation between \( x \) and \( y \) by investing \( k \) amount. To show our results in the simplest way, we assume that firm 2 does not invest in production differentiation, may be due to a prohibitive cost of product differentiation. Hence, we consider firm 1’s non-strategic incentive for investment in product differentiation.\(^6\)

If both firms invest in product differentiation, the analysis will be a bit more complicated due to the presence of strategic and non-strategic incentives for investments in product differentiation. In this situation, as in Lambertini and Rossini (1998), there are several possible equilibria – both firms invest in product differentiation, only one firm invests in product differentiation, and no firm invests in

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\(^6\) A firm’s non-strategic (strategic) benefit from investment in product differentiation is given by its payoff from investment, net of its payoff from no investment, when the competitor firm does not invest (invests).
product differentiation. We avoid this investment game for showing our results in the simplest way, since this investment game does not add much to our main purpose.

We restrict our attention to $c \in [0, 0.3]$ and $g \in [0, 0.7]$. This helps us to prove our results in the simplest way by ensuring positive outputs of both firms under Cournot and Bertrand competition.

We consider the following game. Conditional on the patent system that determines firm 2’s marginal cost of production, firm 1 decides at stage 1 whether or not to invest $k$ to create product differentiation. At stage 2, the firms choose their product-market variable (output or price) simultaneously and the profits are realised. We solve the game through backward induction.

3. Shubik and Levitan type demand function

First, consider a demand function that is due to Shubik and Levitan (1980) where the market size does not depend on the degree of product differentiation. Hence, the role of product differentiation in this situation is only to reduce the intensity of competition between the firms.

3.1. Cournot competition

Straightforward calculation shows that, if firm 1 invests in product differentiation and there is a strong patent protection so that firm 2’s marginal cost is $c$, the equilibrium outputs are $x^* = \frac{4 - (3 - c)g}{(4 - g)(4 - 3g)}$ and $y^* = \frac{4(1 - c) - (3 - 2c)g}{(4 - g)(4 - 3g)}$. The equilibrium profits
of firms 1 and 2 are respectively
\[ \pi_1^* = \frac{(2-g)[4-(3-c)g]^2}{(4-g)^2(4-3g)^2} - k \]
and
\[ \pi_2^* = \frac{(2-g)[4(1-c)-(3-2c)g]^2}{(4-g)^2(4-3g)^2}. \]

If firm 1 does not invest in product differentiation, \(x\) and \(y\) are perfect substitutes (i.e., \(g=1\)) and the profits of firms 1 and 2 are respectively
\[ \pi_1^0 = \frac{(1+c)^2}{9} \]
and
\[ \pi_2^0 = \frac{(1-2c)^2}{9}. \]

Firm 1 invests in product differentiation under strong patent protection if
\[ k < \frac{(2-g)[4-(3-c)g]^2}{(4-g)^2(4-3g)^2} \cdot \frac{(1+c)^2}{9} \equiv k_{1s}. \quad (4) \]

If \(c < .05\) (approx.), \(k_{1s}^c\) is positive for \(g \in [0, 7]\). Otherwise, \(k_{1s}^c\) is not positive for all \(g \in [0, 7]\), suggesting that if \(c\) is not very small, firm 1 may not have the incentive for product differentiation even if there is no cost of product differentiation. If \(c\) is not very small, firm 1 gets a large market share when the products are perfect substitutes. However, product differentiation allows firm 2 to increase its market share significantly by reducing the intensity of competition. Hence, if \(c\) is not very small, firm 1 may not have the incentive for product differentiation.

Now consider a weak patent protection that reduces firm 2’s marginal cost to 0. In this situation, firm 1 invests in product differentiation if
\[ k < \frac{(2-g)}{(4-g)^2} \cdot \frac{1}{9} \equiv k_{1w}. \quad (5) \]

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We get that
\[ k_{1s}^c > 0 \quad \text{if} \quad c < c^*(g), \]
where
\[ c^*(g) = \frac{-256+328g-114g^2+9g^3}{256-256g+78g^2-9g^3} + 12 \sqrt{\frac{512-1280g+1216g^2-544g^3+114g^4-9g^5}{(256-256g+78g^2-9g^3)^2}}. \]
We get that $k_1^{cW} > 0$ for $g \in [0,.7]$\(^8\) and $k_1^{cS} < k_1^{cW}$, suggesting that firm 1’s incentive for product differentiation is higher under weak patent protection. Hence, the following result is immediate.

**Proposition 1:** Consider $c \in [0,.3]$ and $g \in [0,.7]$. Under Cournot competition and a Shubik and Levitan type demand function, firm 1’s incentive for product differentiation is higher under weak patent protection compared to a strong patent protection.

Since a weak patent protection increases the intensity of competition by reducing the marginal cost of firm 2, it increases firm 1’s incentive for product differentiation for reducing the intensity of competition.

Now consider the effects of a weak patent protection on consumer surplus. If firm 2’s marginal cost is $t$, where $t=c$ under strong patent protection and $t=0$ under weak patent protection, consumer surplus is

$CS^{c,I} = \frac{1}{6} \left[ -\frac{t^2}{(4-3g)^2} + \frac{t^2}{4-3g} + \frac{3(2-t)^2}{(4-g)^2} \right]$ under investment by firm 1 and it is

$CS^{c,NI} = \frac{(2-t)^2}{18}$ under no investment by firm 1.

If firm 1 invests or does not invest in product differentiation irrespective of the patent system, a weak patent protection increases consumer surplus compared to a strong patent protection.

\(^8\) Even if there is no cost of product differentiation, firm 1 may not prefer product differentiation under weak patent protection if weak patent protection does not reduce firm 2’s marginal cost significantly. However, a weak patent protection increases firm 1’s incentive for product differentiation since $\frac{\partial k_1^{cS}}{\partial c} < 0$. 


Now consider the situation where firm 1 invests only under weak patent protection. This happens for \( k_i^{cS} < k < k_i^{cW} \). In this situation, consumer surplus is

\[
CS^{c,S,NI} = \frac{(2-c)^2}{18}
\]

under strong patent protection and it is

\[
CS^{c,W,I} = -\frac{2}{(4-g)^2}
\]

under weak patent protection. We get that \( CS^{c,W,I} < CS^{c,S,NI} \) if either \( 0 < c < .18(\text{approx.}) \) or \( .18 < c < .3 \) and \( g < g^*(c) = \frac{2-4c}{2-c} \).

The above result is in contrast to Lahiri and One (1988), where consumer surplus increases if marginal cost of the high-cost firm reduces. Hence, their result suggests that a weak patent protection (which reduces the marginal cost of the high-cost firm) increases consumer surplus if it does not affect innovation. In contrast, we show that even if a weak patent protection does not affect innovation, it may induce the innovator to adopt a competition reducing strategy through higher product differentiation, which reduces consumer surplus.

The reason for the above result is as follows. If firm 1’s decision on product differentiation is not affected by the patent system, a weak patent protection increases consumer surplus by creating production efficiency in the industry. However, if a weak patent protection increases product differentiation, it tends to reduce consumer surplus by reducing the intensity of competition. If either the marginal cost of the non-innovating firm is small (i.e., \( 0 < c < .18 \)) so that cost reduction under weak patent protection does not increases production efficiency significantly or investment by firm 1 creates large product differentiation (i.e., \( g < g^*(c) \)) so that it reduces competition significantly, the competition reducing effect of product differentiation dominates the production efficiency enhancing effect of a marginal cost reduction, and a weak patent protection reduces consumer surplus compared to a strong patent protection.
The above discussion is summarised in the following proposition.

**Proposition 2:** Consider $c \in [0,.3]$, $g \in [0,.7]$, Cournot competition and a Shubik and Levitan type demand function.

(i) If firm 1 invests in product differentiation only under weak patent protection, consumer surplus is higher under a strong patent protection compared to a weak patent protection if either $0 < c < .18$ or $0.18 < c < .3$ and $g < g^*(c) = \frac{2-4c}{2-c}$.

(ii) Consumer surplus is higher under weak patent protection compared to a strong patent protection if firm 1 either invests or does not invest in product differentiation irrespective of the patent system.

Now consider the effects of a weak patent protection on welfare, which is the sum of total net profits and consumer surplus. If firm 2’s marginal cost is $t$, where $t=c$ under strong patent protection and $t=0$ under weak patent protection, welfare is

$$W^{c,I} = \frac{4(1-t)(4-3g)(3-g)+t^2(2-g)(48-48g+11g^2)}{2(4-3g)^2(4-g)^2} - k$$ under investment by firm 1 and

$$W^{c,NI} = \frac{8(1-t)+11t^2}{18}$$ under no investment by firm 1.

As mentioned above, since a weak patent protection in our analysis eliminates the marginal cost difference between the firms, it eliminates the effects shown in Lahiri and Ono (1988), and a weak patent protection increases welfare compared to a strong patent protection if firm 1 either invests or does not invest in product differentiation irrespective of the patent system.
Now consider the situation where firm 1 invests in product differentiation only under weak patent protection, which occurs for $k_i^{cS} < k < k_i^{cW}$. In this situation, welfare is $W^{c,s,N} = \frac{8(1-c)+11c^2}{18}$ under strong patent protection but it is

$$W^{c,w,l} = \frac{2(3-g)}{(4-g)^2} - k$$

under weak patent protection. We get that welfare can be lower under weak patent protection even if we evaluate it at $k = k_i^{cS}$. For example, $k_i^{cS} = 0$ if $c = 0.1$. We get in this situation that $W^{c,w,l}$ is lower than $W^{c,s,N}$ if $g < 0.47$. The reason for a lower welfare under weak patent protection is similar to the reason for a lower consumer surplus under weak patent protection.

The following proposition summarises the above discussions on welfare.

**Proposition 3:** Consider $c \in [0, 0.3], \ g \in [0, 0.7], \ Cournot \ competition \ and \ a \ Shubik \ and \ Levitan \ type \ demand \ function.

(i) A strong patent protection may create higher welfare compared to a weak patent protection if firm 1 invests in product differentiation only under weak patent protection.

(ii) Welfare is higher under weak patent protection compared to a strong patent protection if firm 1 either invests or does not invest in product differentiation irrespective of the patent system.

3.2. Bertrand competition

Now consider the situation under Bertrand competition. Straightforward calculation shows that, if firm 1 invests in product differentiation and there is a strong patent protection so that firm 2’s marginal cost is $c$, the equilibrium prices are
\[ P_x^* = \frac{8-10g + 4cg - 2cg^2 + 2g^2}{(4-g)(4-3g)} \quad \text{and} \quad P_y^* = \frac{8-10g + 2g^2 + cg^2}{(4-g)(4-3g)}. \]

The equilibrium outputs and profits of firms 1 and 2 are respectively

\[ x^* = \frac{(2-g)(4-5g + 2cg + g^2 - cg^2)}{2(4-g)(4-3g)(1-g)} \quad \text{and} \quad y^* = \frac{(2-g)(4-8c - 5g + 8cg + g^2 - cg^2)}{2(4-g)(4-3g)(1-g)}, \]

and

\[ \pi_1^* = \frac{(2-g)(4-5g + 2cg + g^2 - cg^2)^2}{(4-g)^2(4-3g)^2(1-g)} - k \]

and

\[ \pi_2^* = \frac{(2-g)(4-8c - 5g + 8cg + g^2 - cg^2)^2}{(4-g)^2(4-3g)^2(1-g)}. \]

If firm 1 does not invest in product differentiation, \( x \) and \( y \) are perfect substitutes (i.e., \( g=1 \)) and the profits of firms 1 and 2 are respectively \( \pi_1^0 = c(1-c) \) and \( \pi_2^0 = 0 \).

Firm 1 invests in product differentiation under strong patent protection if

\[ k < \frac{(2-g)(4-5g + 2cg + g^2 - cg^2)^2}{(4-g)^2(4-3g)^2(1-g)} - c(1-c) \equiv k_1^{bs}. \]

If \( c < .14 (\text{approx}) \), \( k_1^{bs} \) is positive for \( g \in [0, .7] \). Otherwise, \( k_1^{bs} \) is not positive for all \( g \in [0, .7] \), suggesting that if \( c \) is not very small, firm 1 may not have the incentive for product differentiation even if there is no cost of product differentiation. Although firm 1 faces intensive competition from firm 2 if the goods are perfect substitutes, it is the only firm that sells the product. Hence, if firm 1 does not invest, it gets a restricted monopoly profit. On the other hand, if there is product differentiation, it helps to increase the price but allows firm 2 to share the market demand. Hence, if \( c \) is not
very small, so that firm 1’s profit under no product differentiation is significant, firm 1 may not have the incentive for product differentiation.

Now consider a weak patent protection where \( c=0 \). Firm 1 invests in product differentiation in this situation if

\[
k < \frac{(2-g)(1-g)}{(4-3g)^2} \equiv k_i^{bw}.
\]

We get that \( k_i^{bw} > 0 \) for \( g \in [0,.7] \)\(^{10} \) and \( k_i^{bs} < k_i^{bw} \), suggesting that firm 1’s incentive for product differentiation is higher under weaker patent protection. Hence, the following result is immediate.

**Proposition 4:** Consider \( c \in [0,.3] \) and \( g \in [0,.7] \). Under Bertrand competition and a Shubik and Levitan type demand function, firm 1’s incentive for product differentiation is higher under weak patent protection compared to a strong patent protection.

The reason for Proposition 4 is similar to that of Proposition 1.

Now consider the effects of a weak patent protection on consumer surplus. If firm 2’s marginal cost is \( t \), where \( t=c \) under strong patent protection and \( t=0 \) under weak patent protection, consumer surplus is

\[
c^{**}(g) = \frac{1}{20} \left( 11 - \frac{10}{(256 - g(768 + g(-872 + g(460 + g(-111+10g))))^2)} \right).
\]

\(^9\) We get that \( k_i^b > 0 \) if \( c < c^{**}(g) \), where \( c^{**}(g) \) is given above.

\(^{10}\) Even if there is no cost of product differentiation, firm 1 may not prefer product differentiation under weak patent protection if weak patent protection does not reduce firm 2’s marginal cost significantly.
If firm 1 invests or does not invest irrespective of the patent system, a weak patent protection increases consumer surplus compared to a strong patent protection.

Now consider the situation where firm 1 invests in product differentiation only under weak patent protection, which occurs for \( k_1^{bs} < k < k_1^{bw} \). In this situation, consumer surplus is \( CS^{b,S,NI} = \frac{(1-t)^2}{2} \) under strong patent protection and it is \( CS^{b,W,NI} = \frac{(2-g)^2}{2(4-3g)^2} \) under weak patent protection. We get that consumer surplus is higher under strong patent protection for \( c \in [0,.3] \) and \( g \in [0,.7] \). Although a weak patent protection increases production efficiency by reducing firm 2’s marginal cost of production, it also reduces product-market competition by inducing product differentiation by firm 1. We find that the effect of product differentiation dominates the effect of marginal cost reduction and consumer surplus is higher under strong patent protection than under weak patent protection.

**Proposition 5:** Consider \( c \in [0,.3], \ g \in [0,.7], \) Bertrand competition and a Shubik and Levitan type demand function.

However, a weak patent protection increases firm 1’s incentive for product differentiation since \( \frac{\partial k_1^{bs}}{\partial c} < 0 \).
(i) If firm 1 invests in product differentiation only under weak patent protection, consumers surplus is higher under strong patent protection compared to a weak patent protection for \( c \in [0,0.3] \) and \( g \in [0,0.7] \).

(ii) Consumer surplus is higher under weak patent protection compared to a strong patent protection if firm 1 either invests or does not invest in product differentiation irrespective of the patent system.

The comparison between Propositions 2 and 5 shows that if we consider a Shubik and Levitan type demand function and firm 1 invests in product differentiation only under weak patent protection, the possibility of higher consumer surplus under strong patent protection compared to a weak patent protection is more under Bertrand competition than under Cournot competition. This happens since the negative effect of a higher product differentiation (relative to the positive effect of a marginal cost reduction) on consumer surplus is higher under Bertrand competition compared to Cournot competition.

Now consider the welfare effects of a weak patent protection. If firm 2’s marginal cost is \( t \), where \( t=c \) under strong patent protection and \( t=0 \) under weak patent protection, welfare \( W^{b,l} \) is

\[
W^{b,l} = \frac{(2-g)[(4-g)^2(1-g)(6-5g)-t(4-g)^2(1-g)(8-7g)]}{2(4-g)^2(4-3g)^2(1-g)} - k \quad \text{under investment by firm 1 and } W^{b,NI} = \frac{(1-t^2)}{2} \quad \text{under no investment by firm 1.}
\]

A weak patent protection increases welfare compared to a strong patent protection if firm 1 either invests or does not invest in product differentiation.
Now consider the situation where firm 1 invests in product differentiation only under weak patent protection, which occurs for \( k_1^{bs} < k < k_1^{bw} \). In this situation, welfare is \( W^{c,s,nl} = \frac{(1-c^2)}{2} \) under strong patent protection but it is \( W^{c,w,l} = \frac{(2-g)(6-5g)}{2(4-3g)^2} - k \) under weak patent protection. We get that welfare can be lower under weak patent protection even if we evaluate it at \( k = k_1^{bs} \). For example, \( k_1^{bs} > 0 \) for \( g \in [0, .7] \) if \( c = .1 \). In this situation, \( W^{c,w,l} \) is lower than \( W^{c,s,nl} \) for \( g \in [0, .7] \). The reason for a lower welfare under weak patent protection is similar to the reason for a lower consumer surplus under weak patent protection.

Like the effects on consumer surplus, we get that the possibility of a higher welfare under strong patent protection is more under Bertrand competition compared Cournot competition. For example, our example with \( c = .1 \) shows that, if there is Bertrand competition, welfare is higher under strong patent protection for \( g \in [0, .7] \), but, under Cournot competition, welfare is higher under strong patent protection for \( g < .47 \).

The following proposition summarises the above discussion on welfare.

**Proposition 6:** Consider \( c \in [0, .3] \), \( g \in [0, .7] \), Cournot competition and a Shubik and Levitan type demand function.

(i) A strong patent protection may create higher welfare compared to a weak patent protection if firm 1 invests in product differentiation only under weak patent protection.
(ii) Welfare is higher under weak patent protection compared to a strong patent protection if firm 1 either invests or does not invest in product differentiation irrespective of the patent system.

The discussions in this section show that if there is a weak patent protection, it may induce the innovator to adopt a competition reducing strategy through product differentiation. We show that if the innovator invests in product differentiation only under weak patent protection, the competition reducing effect of product differentiation may dominate the production efficiency enhancing effect of a marginal cost reduction, and may reduce consumer surplus and welfare under weak patent protection compared to a strong patent protection. While product differentiation reduces competition between the firms, it may also increase the total market size. The demand function considered by Shubik and Levitation (1980) does not consider the market expansion effect of product differentiation, while the market expansion effect of production differentiation is found in the demand function considered by Bowley (1924). It is intuitive that if product differentiation creates a positive effect on the consumers and on the society by increasing the total market size, it reduces the incentive for a strong patent protection as shown in this section. We formally show in the next section the effects of the patent system under a Bowley type demand function.

4. Bowley type demand function

We consider in this section a demand function that is due to Bowley (1924). As in the previous section, we restrict our attention to $c \in [0, .3]$ and $g \in [0, .7]$.

4.1. Cournot competition
Straightforward calculation shows that, if firm 1 invests in product differentiation and there is a strong patent protection, the equilibrium outputs are \( x^* = \frac{2 - g + cg}{4 - g^2} \) and \( y^* = \frac{2 - 2c - g}{4 - g^2} \). The equilibrium profits of firms 1 and 2 are respectively

\[
\pi_1^* = \frac{(2 - g + cg)^2}{(4 - g^2)^2} - k \quad \text{and} \quad \pi_2^* = \frac{(2 - 2c - g)^2}{(4 - g^2)^2}.
\]

If firm 1 does not invest in product differentiation, \( x \) and \( y \) are perfect substitutes and the profits of firms 1 and 2 are respectively \( \pi_1^0 = \frac{(1 + c)^2}{9} \) and \( \pi_2^0 = \frac{(1 - 2c)^2}{9} \).

Firm 1 invests in product differentiation under strong patent protection if

\[
k < \frac{(2 - g + cg)^2}{(4 - g^2)^2} - \frac{(1 + c)^2}{9} \equiv k_1^{c,s}.
\]

If \( c < .27(\text{approx.}) \), \( k_1^{c,s} \) is positive for \( g \in [0,.7] \). Otherwise, \( k_1^{c,s} \) is not positive for all \( g \in [0,.7] \), suggesting that if \( c \) is not very small, firm 1 may not have the incentive for product differentiation even if there is no cost of product differentiation. The reason for this result is similar to the reason mentioned in the previous section.

Now consider a weak patent protection that reduces firm 2’s marginal cost to 0. Firm 1 invests in product differentiation under weak patent protection if

\[
k < \frac{5 - 4g - g^2}{9(2 + g)^2} \equiv k_1^{c,w}.
\]

\[\text{\(11\) We get that } k_1^{c,s} > 0 \text{ if } c < c(g), \text{ where } c(g) = \frac{2 - g}{4 + g}.\]
We get that \( k_1^{cw} > 0 \) for \( g \in [0, 7] \)\(^{12} \) and \( k_1^{cs} < k_1^{cw} \). Hence, the following result is immediate.

**Proposition 7:** Consider \( c \in [0, 3] \) and \( g \in [0, 7] \). Under Cournot competition and a Bowley type demand function, firm 1’s incentive for product differentiation is higher under weak patent protection compared to a strong patent protection.

Although we are now considering a demand function where higher product differentiation increases the total market size, the reason for the above result is similar to Proposition 1.

Now consider the effects of a weak patent protection on consumer surplus. If firm 2’s marginal cost is \( t \), where \( t = c \) under strong patent protection and \( t = 0 \) under weak patent protection, consumer surplus is

\[
CS^{c,I} = \frac{(4 - 3g^2)(2 - 2t + t^2) + 2(1 - t)g^3}{2(4 - g^2)^2}
\]

under investment by firm 1 and \( CS^{c,NI} = \frac{(2 - t)^2}{18} \) under no investment by firm 1.

If firm 1 either invests or does not invest irrespective of the patent system, a weak patent protection increases consumer surplus compared to a strong patent protection.

Now consider the situation where firm 1 invests only under weak patent protection, which occurs for \( k_1^{cs} < k < k_1^{cw} \). In this situation, consumer surplus is

\(^{12}\) Even if there is no cost of product differentiation, firm 1 may not prefer product differentiation under weak patent protection if weak patent protection does not reduce firm 2’s marginal cost significantly. However, a weak patent protection increases firm 1’s incentive for product differentiation since \( \frac{\partial k_1^{cs}}{\partial c} < 0 \).
\[ CS^{c,S,NI} = \frac{(2-c)^2}{18} \] under strong patent protection and it is \[ CS^{c,W,I} = \frac{1+g}{(2+g)^2} \] under weak patent protection. We get that \( CS^{c,W,I} > CS^{c,S,NI} \). This result is in contrast to the case under a Shubik and Levitan type demand function. The market expansion effect under a Bowley type demand function following product differentiation is the reason for this difference. If firm 1 invests in product differentiation only under weak patent protection, on the one hand, it tends to reduce consumer surplus by reducing competition, but, on the other hand, it tends to increase consumer surplus by increasing the total market size. The market expansion effect of product differentiation along with the production efficiency enhancing effect of a lower marginal cost dominates the competition reducing effect of product differentiation under weak patent protection, thus increasing consumer surplus under weak patent protection compared to a strong patent protection.

We get the following proposition from above discussion.

**Proposition 8:** Consider \( c \in [0,0.3] \) and \( g \in [0,0.7] \). Under Cournot competition and a Bowley type demand function, consumer surplus is higher under weak patent protection compared to a strong patent protection, irrespective of the effect of the patent system on product differentiation.

Now consider the effects of a weak patent protection on welfare. If the degree of product differentiation is \( g \) and firm 2’s marginal cost is \( t \), where \( t=c \) under strong patent protection and \( t=0 \) under weak patent protection, welfare is
A weak patent protection increases welfare compared to a strong patent protection if firm 1 either invests or does not invest in product differentiation irrespective of the patent system.

Now consider the situation where firm 1 invests in product differentiation only under weak patent protection, which occurs for \( k_{1s}^c < k < k_{1w}^c \). In this situation, welfare is \( W^{c,s,NI} = \frac{8(1-c)+11c^2}{18} \) under strong patent protection but it is \( W^{c,w,I} = \frac{(3+g)}{(2+g)^2} - k \) under weak patent protection. We get that \( W^{c,w,I} > W^{c,s,NI} \) even if we evaluate it at \( k = k_{1w}^c \). This is in contrast to the result derived under a Shubik and Levitan demand function. The market expansion effect of product differentiation under a Bowley type demand function is responsible for creating higher welfare under weak patent protection even if firm 1 invests in product differentiation only under weak patent protection.

We get the following proposition from above discussion.

**Proposition 9:** Consider \( c \in [0,.3] \) and \( g \in [0,.7] \). Under Cournot competition and a Bowley type demand function, welfare is higher under weak patent protection compared to a strong patent protection, irrespective of the effect of the patent system on product differentiation.
4.2. Bertrand competition

Now consider Bertrand competition in the product market. Straightforward calculation shows that, if firm 1 invests in product differentiation and there is a strong patent protection, the equilibrium prices are

\[ P_1^* = \frac{2 - g + cg - g^2}{4 - g^2} \quad \text{and} \quad P_2^* = \frac{2c - g - g^2}{4 - g^2}. \]

The equilibrium outputs and profits of firms 1 and 2 are respectively

\[ x^* = \frac{2 - g + cg - g^2}{(4 - g^2)(1 - g^2)} \quad \text{and} \quad y^* = \frac{2 - 2c - g - g^2 + cg^2}{(4 - g^2)(1 - g^2)}, \quad \text{and} \quad \pi_1^* = \frac{(2 - g + cg - g^2)^2}{(4 - g^2)^2(1 - g^2)} - k \]

and

\[ \pi_2^* = \frac{(2 - 2c - g - g^2 + cg^2)^2}{(4 - g^2)^2(1 - g^2)}. \]

If firm 1 does not invest in product differentiation, \( x \) and \( y \) are perfect substitutes and the profits of firms 1 and 2 are respectively \( \pi_1^0 = c(1 - c) \) and \( \pi_2^0 = 0 \).

Firm 1 invests in product differentiation under strong patent protection if

\[ k < \frac{(2 - g + cg - g^2)^2}{(4 - g^2)^2(1 - g^2)} - c(1 - c) \equiv k_{1s}^{b}. \quad (10) \]

If \( c < .16(\text{approx.}), \) \( k_{1s}^{b} \) is positive for \( g \in [0,.7]. \)\textsuperscript{13} Otherwise, \( k_{1s}^{b} \) is not positive for all \( g \in [0,.7], \) suggesting that if \( c \) is not very small, firm 1 may not have the incentive for product differentiation even if there is no cost of product differentiation.

Now consider a weak patent protection that reduces firm 2’s marginal cost to 0. Firm 1 invests in product differentiation under weak patent protection if

\[ k < \frac{1 - g}{(2 - g)^2(1 + g)} \equiv k_{1w}^{b}. \quad (11) \]

\textsuperscript{13} We get that \( k_{1s}^{b} > 0 \) if \( c < c(g), \) where

\[ c(g) = \frac{1}{2} \left( 1 - \sqrt{\frac{g(2 + g)(2 - 3g + g^2)^2(4 + 2g - 3g^2 + g^4)}{(16 - 23g^2 + 9g^4 - g^6)^2} - \frac{g(4 - g - 2g^2)}{16 - 23g^2 + 9g^4 - g^6}} \right). \]
We get that $k_1^{b \cdot w} > 0$ for $g \in [0, .7]$ and $k_1^{b \cdot S} < k_1^{b \cdot W}$. Hence, the following result is immediate.

**Proposition 10:** Consider $c \in [0, .3]$ and $g \in [0, .7]$. Under Bertrand competition and a Bowley type demand function, firm 1’s incentive for product differentiation is higher under weak patent protection compared to a strong patent protection.

The reason for above proposition is similar to Proposition 7.

Now consider the effects of a weak patent protection on consumer surplus. If firm 2’s marginal cost is $t$, where $t=a$ under strong patent protection and $t=0$ under weak patent protection, consumer surplus is

$$CS_{b \cdot I} = \frac{(4 - 3g^2)(2 - 2t + t^2) - 2(1-t)g^3}{2(4-g^2)^2(1-g^2)}$$

under investment by firm 1 and $CS_{c \cdot NI} = \frac{(1-t)^2}{2}$ under no investment by firm 1.

If firm 1 either invests or does not invest in product differentiation irrespective of the patent system, a weak patent protection increases consumer surplus compared to a strong patent protection.

Now consider the situation where firm 1 invests only under weak patent system, which occurs for $k_1^{b \cdot S} < k < k_1^{b \cdot W}$. In this situation, consumer surplus is

$$CS_{b \cdot S, NI} = \frac{(1-c)^2}{2}$$

under strong patent protection and it is $CS_{b \cdot W, I} = \frac{1}{(2-g)^2(1+g)}$ under weak patent protection. We get that $CS_{c \cdot W, I} < CS_{c \cdot S, NI}$ if either

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14 Even if the cost of product differentiation is zero, firm 1 may not prefer product differentiation under weak patent protection if weak patent protection does not reduce firm 2’s marginal cost significantly. However, a weak patent protection increases firm 1’s incentive for product differentiation since $\frac{\partial k_{1 \cdot S}}{\partial c} < 0$. 

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0 < c < 0.16(approx.) or 0.16 < c < 0.3 and g < g(c), where

\[
\frac{(1-c)^2}{2} = \frac{1}{(2-g(c))^2(1+g(c))}.
\]

The following proposition summarises the discussion on consumer surplus.

**Proposition 11:** Consider \( c \in [0,0.3], g \in [0,0.7] \), Bertrand competition and a Bowley type demand function.

(i) If firm 1 invests in product differentiation only under weak patent protection, consumer surplus is higher under strong patent protection compared to a weak patent protection if either \( 0 < c < 0.16(approx.) \) or \( 0.16 < c < 0.3 \) and \( g < g(c) \), where

\[
\frac{(1-c)^2}{2} = \frac{1}{(2-g(c))^2(1+g(c))}.
\]

(ii) Consumer surplus is higher under weak patent protection compared to a strong patent protection if firm 1 either invests or does not invest in product differentiation irrespective of the patent system.

Now look at the welfare effects of a weak patent protection. If firm 2’s marginal cost is \( t \), where \( t=c \) under strong patent protection and \( t=0 \) under weak patent protection, welfare is

\[
W^{b,I} = \frac{2(1-t)(1-g)(2+g)^2(3-2g) + t^2(12-9g^2 + 2g^4)}{2(4-g^2)^2(1-g^2)} - k \text{ under investment by firm 1}
\]

and

\[
W^{b,NI} = \frac{(1-t^2)}{2} \text{ under no investment by firm 1}.
\]
A weak patent protection increases welfare compared to a strong patent protection if firm 1 either invests or does not invest in product differentiation irrespective of the patent system.

Now consider the situation where firm 1 invests in product differentiation only under weak patent protection, which occurs for $k_1^{bS} < k < k_1^{bW}$. In this situation, welfare is $W_{c,S,NI} = \frac{(1-c^2)}{2}$ under strong patent protection but it is $W_{c,W,I} = \frac{3-2g}{(2-g)^2(1+g)} - k$ under weak patent protection. We get that $W_{c,W,I}$ can be higher or lower than $W_{c,S,NI}$ for $k_1^{bS} < k < k_1^{bW}$.

The following proposition summarises the above discussion on welfare.

**Proposition 12:** Consider $c \in [0, .3], \ g \in [0, .7], Bertrand competition and a Bowley type demand function.

(i) Welfare may be higher or lower under strong patent protection compared to a weak patent protection if firm 1 invests in product differentiation only under weak patent protection.

(ii) Welfare is higher under weak patent protection compared to a strong patent protection if firm 1 either invests or does not invest in product differentiation irrespective of the patent system.

5. Conclusion

It is generally believed that a weak patent protection makes the consumers and the society better off if the patent system does not affect innovation. We show that this conclusion may not be valid and there can be a case for a strong patent protection even
if the patent system does not affect innovation. A weak patent protection may induce an innovator to adopt competition reducing strategies such as product differentiation, which may help to reduce consumer surplus and welfare. The following table summarises the effects of the patent system on consumer surplus and welfare when the innovator invests in product differentiation only under weak patent protection. In table 1, we use the terms $\Delta CS$ to imply “Consumer surplus (strong patent) – Consumer surplus (weak patent)” and $\Delta W$ to imply “Welfare (strong patent) – Welfare (weak patent)”. The results show that the type of product-market competition and the type of demand function play important roles in determining the effects of the patent system on consumer surplus and welfare. For example, consumer surplus and welfare are lower under strong patent protection if we consider a Bowley type demand function and Cournot competition. However, if we consider either Bertrand competition and a Bowley type demand function or Cournot competition and a Shubik and Levitan type demand function, consumer surplus and welfare can be higher under strong patent protection.
<table>
<thead>
<tr>
<th>Demand Function</th>
<th>Competition</th>
<th>$\Delta CS$</th>
<th>$\Delta W$</th>
</tr>
</thead>
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<tr>
<td>Shubik and Levitan</td>
<td>Cournot Competition</td>
<td>$\Delta CS &gt; 0$ if $0 &lt; c &lt; 0.18$ (approx.) or $0.18 &lt; c &lt; 0.3$ and $g &lt; g^*\left(c\right) = \frac{2 - 4c}{2 - c}$</td>
<td>$\Delta W \geq 0 &lt; \Delta W$</td>
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<tr>
<td></td>
<td>Bertrand Competition</td>
<td>$\Delta CS &gt; 0$ for $c \in [0, 0.3]$ and $g \in [0, 0.7]$</td>
<td>$\Delta W \geq 0 &lt; \Delta W$</td>
</tr>
<tr>
<td>Bowley type</td>
<td>Cournot Competition</td>
<td>$\Delta CS &lt; 0$ for $c \in [0, 0.3]$ and $g \in [0, 0.7]$</td>
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<tr>
<td>demand function</td>
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<tr>
<td></td>
<td>Bertrand Competition</td>
<td>$\Delta CS &gt; 0$ if $0 &lt; c &lt; 0.16$ (approx.) or $0.16 &lt; c &lt; 0.3$ and $g &lt; g(c)$, where [ \frac{(1 - c)^2}{2} = \frac{1}{(2 - g)^3(1 + g)} ]</td>
<td>$\Delta W \geq 0 &lt; \Delta W$</td>
</tr>
</tbody>
</table>

Table 1: The effects of the patent system on consumer surplus and welfare when the innovator invests in product differentiation only under weak patent protection.
References


