Patent pool under endogenous technology choice.

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Abstract: It is generally believed that patent pools by complementary input suppliers make the consumers, final goods producers and the society better off by reducing the complements problem. We show that this may not be the case under endogenous technology choice. Although a patent pool reduces input price, it may make the consumers and the society worse off by reducing innovation. We also show that a patent pool makes the input suppliers better off, but it may not make all final goods producers better off compared with non-cooperation between the input suppliers.

Key words: Complementary inputs; Patent pool; Innovation; Welfare

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1. Introduction

Many products use complementary inputs produced by different firms, which often cooperate or pool their patents. For example, successful patent pools can be found between the patent holders of MPEG-2, DVD and 3G Platform. Detail information about some of the successful patent pools can be found in Aoki and Nagaoka (2004). According to Clarkson (2003), sales in 2001 of the devices based in (whole or part) pooled patents are at least $100 billion.

Considering perfectly complementary patents, Shapiro (2000) shows that patent pool solves the problems of complementary effects. While choosing prices, the patent holders do not internalise the negative external effects of their pricing on other patent holders’ revenues, thus creating the complements problem. Cooperation between the patent holders or patent pool solves the complements problem, and increases profits of the patent holders and also makes the consumers better off by reducing the price of the final goods. Lerner and Tirole (2004) show that patent pool increases welfare if the patents are complements, while it reduces welfare if the patents are substitutes.¹

We show that the general belief about the positive effects of patent pools by complementary input suppliers on the consumers, final goods producers and the society may not hold true under endogenous technology choice. Although a patent pool reduces input price, it may make the consumers and the society worse off by reducing innovation. Using a simple model with innovating and non-innovating firms² using perfectly complementary inputs, we show that a patent pool may decrease (increase) the incentive for innovation if the technological improvement through R&D is not very small (small). Hence, although a patent pool solves the

¹ It is argued that overlapping intellectual property rights create the problem in commercialising new innovations. See, Gallini (2002) for an overview of this literature. The proposal for patent pool by Priest (1977), Merges (1999) and Shapiro (2000) is a way to resolve this problem. Gilbert (2004) provides an overview of patent pools.

complements problem, it may either substitute or complement innovation by the final goods producer. The adverse effect of a patent pool on innovation may dominate its beneficial input price effect, and a patent pool may make the consumers worse off compared to non-cooperation. We show that if the product market is very competitive and the technological improvement through R&D is large, patent pool also reduces social welfare by reducing innovation. We also show that a patent pool makes the input suppliers better off, but it may reduce profit of the technologically superior final goods producer if the technological difference between the final goods producers is sufficiently large.

The economic analyses on patent pools are increasing. Aoki and Nagaoka (2004) show that patent pools may create the free rider problem. Kim (2004) and Schmidt (2008) show the effects of vertical integration in determining the impacts of a patent pool. Gilbert and Katz (2010) show the effects of different sharing rules in patent pools on the patent holders’ incentives for innovations. Choi (2010) shows the incentive for a patent pool in the presence of uncertainty and coverage of patents. Lerner et al. (2007) empirically examine licensing rules in the patent pools. The empirical analysis by Layne-Farrar and Lerner (2011) shows that vertically integrated firms are more likely to create patent pools, and the holders of valuable patents may less likely to create patent pools depending on the profit sharing rule in the pool. Kato (2004) considers the implications of a patent pool of substitute patents. While these papers provide important insights, they ignored the dynamic effect of a patent pool due to its impact on the final goods producer’s innovation, which, like a patent pool by complementary input suppliers, helps to reduce the innovating final goods producer’s marginal cost of production. This paper is a step to fill this gap in the literature.

The remainder of the paper is organised as follows. Section 2 describes the model and derives the results. Section 3 concludes.
2. The model and the results

Assume that there are one innovating firm (firm 1) and \((n - 1)\) non-innovating firms (firms 2, ..., \(n\)), where \(n \geq 2\). These firms compete like Cournot oligopolists with producing homogeneous goods, facing the inverse market demand function \(P = 1 - q\), where \(P\) is the price and \(q\) is the total output. Production of the final goods requires two complementary inputs \(x\) and \(y\), patents of which are owned by firm \(X\) and firm \(Y\) respectively. Firms \(X\) and \(Y\) supply the inputs \(x\) and \(y\) respectively to the final goods producers. We assume that the inputs \(x\) and \(y\) are perfect complements. The marginal costs of production for both the inputs are \(c\), which are assumed to be zero, for simplicity. To start with, we assume that all firms require one unit of each input to produce the final goods. However, the innovating firm, firm 1, can invest \(k\) amount in R&D to reduce its input coefficients for both inputs to \(s\), where \(s < 1\). In order to ensure that the input suppliers always supply inputs to all final goods producers, we assume that \(\frac{2}{n + 2} \leq s\) (see Appendix for details). Hence, we consider that \(\frac{2}{n + 2} \leq s < 1\). We consider the binary choice for firm 1’s R&D decision to prove our point in the simplest way. As we discussed below, our main result will hold even if firm 1 could choose the extent of technological improvement through R&D.

Following the previous papers on patent pools (see, e.g., Shapiro, 2000, Lerner and Tirole, 2003, Aoki and Nagaoka, 2004, Schmidt, 2008, Choi, 2010, to name a few), we consider that the input suppliers use non-discriminatory linear prices, which can be supported by the empirical evidence. Layne-Farrar and Lerner (2011) show that linear prices were used by all the patent pools they investigated.

We will consider two types of pricing strategies of the input suppliers. First, firms \(X\) and \(Y\) determine their prices simultaneously to maximise their own profits, which will be called non-cooperation. Second, firms \(X\) and \(Y\) choose their prices to maximise the total profits of firms \(X\) and \(Y\), which will be defined as a patent pool (see, Shapiro, 2000).
We consider the following game. Conditional on non-cooperation or patent pool in the input market, at stage 1, firm 1 decides whether to innovate or not. At stage 2, the input prices are determined by firms \( X \) and \( Y \). At stage 3, the final goods producers (firms \( 1, \ldots, n \)) determine their outputs simultaneously, and the profits are realised. We solve the game through backward induction.

2.1. Non-cooperation in the input market

If firms \( X \) and \( Y \) charge \( w_x \) and \( w_y \) respectively as the per-unit input prices of \( x \) and \( y \), firm 1 and the \( i \)th firm, \( i = 2, \ldots, n \), determine their outputs by maximising the following expressions respectively:

\[
\begin{align*}
\max_{q_1} [1 - q - t(w_x + w_y)] q_1 - f &= (1) \\
\max_{q_i} [1 - q - (w_x + w_y)] q_i, &= (2)
\end{align*}
\]

where \( t = 1 \) and \( f = 0 \) under no innovation by firm 1, and \( t = s \in [0, 1) \) and \( f = k \) under innovation by firm 1.

The equilibrium outputs of firms 1 and the \( i \)th firm, \( i = 2, \ldots, n \), can be found as

\[
q_1 = \frac{1 - nt(w_x + w_y) + (n - 1)(w_x + w_y)}{n+1} \quad \text{and} \quad q_i = \frac{1 - 2(w_x + w_y) + t(w_x + w_y)}{n+1}.
\]

The input demand faced by firms \( X \) and \( Y \) is \( q_x = q_y = tq_1 + \sum_{i=2}^{n} q_i \). Firms \( X \) and \( Y \) determine their prices by maximising the following expressions:

\[
\begin{align*}
\max_{w_x} w_x (tq_1 + \sum_{i=2}^{n} q_i) &= (4) \\
\max_{w_y} w_y (tq_1 + \sum_{i=2}^{n} q_i) &= (5)
\end{align*}
\]

The equilibrium input prices are
\[ w_x^{nc} = w_y^{nc} = \frac{n-1+t}{6(t-1) + 3n(2-2t+t^2)}. \] (6)

If \( t < 1 \), the equilibrium input prices fall as the number of firms (i.e., \( n \)) increases.\(^3\) If \( t < 1 \), more firms increase the elasticity of the input demand function and reduce the equilibrium input prices.

We obtain from (3) and (6) that the equilibrium outputs of firm 1 and firm \( i, i=2,\ldots,n \), are

\[ q_1^{nc} = \frac{-4(1-t) + 2n^2(1-t) + n(2-2t+t^2)}{(n+1)[-6(1-t) + 3n(2-2t+t^2)]} \quad \text{and} \quad q_i^{nc} = \frac{-2(1-t^2) + n(2-4t+3t^2)}{(n+1)[-6(1-t) + 3n(2-2t+t^2)]} \]

respectively. The total equilibrium outputs are

\[ q^{nc} = q_1^{nc} + \sum_{i=2}^{n} q_i^{nc} = \frac{-2n(1-t) - 2(1-t)^2 + n^2(4-6t+3t^2)}{(n+1)[-6(1-t) + 3n(2-2t+t^2)]}. \]

The equilibrium profits of firm 1 and firm \( i, i=2,\ldots,n \), are \( \pi_1^{nc} = (q_1^{nc})^2 - f \) and \( \pi_i^{nc} = (q_i^{nc})^2 \) respectively.

**Proposition 1:** If the input suppliers determine the input prices non-cooperatively, firm 1 invests in R&D for \( k < \left[ \frac{-4(1-s) + 2n^2(1-s) + n(2-2s+s^2)}{(n+1)[-6(1-s) + 3n(2-2s+s^2)]} \right]^2 - \frac{1}{9(n+1)^2} \equiv k^{nc}. \)

**Proof:** If the input suppliers charge the input prices non-cooperatively, firm 1’s profit under innovation is \( \pi_1^{nc,rd} = \left[ \frac{-4(1-s) + 2n^2(1-s) + n(2-2s+s^2)}{(n+1)[-6(1-s) + 3n(2-2s+s^2)]} \right]^2 - k \), while its profit under no innovation is \( \pi_i^{nc,nd} = \frac{1}{9(n+1)^2} \). Firm 1 innovates if \( \pi_1^{nc,rd} > \pi_1^{nc,nd} \), which gives the result. \( \blacksquare \)

The expression \( k^{nc} \) shows firm 1’s maximum willingness to pay for innovation if the input suppliers choose the input prices non-cooperatively. Innovation increases firm 1’s

\[^3\text{We get that } \frac{\partial w_x^{nc}}{\partial n} = \frac{\partial w_y^{nc}}{\partial n} = \frac{-(2-t)(1-t)t}{3[-2(1-t) + n(2-2t+t^2)]^2} < 0 \]
product-market profit compared to no innovation. However, innovation also imposes a cost on firm 1. Hence, firm 1 innovates if the cost of innovation is not very high.

2.2. Patent pool

Now we consider the situation where firms $X$ and $Y$ cooperate or pool their patents and determine the input prices to maximise the total profits of firms $X$ and $Y$.

If the patent pool charges $w_x$ and $w_y$ as the input prices of $x$ and $y$ respectively, the equilibrium outputs of firms 1 and 2 are given by (3) and the demand for inputs is given by $q_x = q_y = tq_1 + \sum_{i=2}^{n} q_i$. The input prices are determined by maximising the following expression:

$$\text{Max}(w_x + w_y)(tq_1 + \sum_{i=2}^{n} q_i).$$

(7)

The equilibrium input prices are

$$w_x^p = w_y^p = \frac{n-1+t}{-8(1-t) + 4n(2 - 2t + t^2)}. \quad (8)$$

Like non-cooperation, we get under patent pool that, if $t < 1$, as the number of firms (i.e., $n$) increases, the elasticity of the input demand function increases and the equilibrium input prices fall. 4

We obtain from (3) and (8) that the equilibrium outputs of firm 1 and firm $i$, $i = 2,...,n$, are

$$q_x^p = \frac{-3(1-t) + n^2(1-t) + n(2 - 2t + t^2)}{(n+1)(-4(1-t) + 2n(2 - 2t + t^2))} \quad \text{and} \quad q_i^p = \frac{-(2-t + t^2) + n(2 - 3t + 2t^2)}{(n+1)(-4(1-t) + 2n(2 - 2t + t^2))}$$

respectively. The total equilibrium outputs are

$$\frac{\partial w_x^p}{\partial n} = \frac{\partial w_y^p}{\partial n} = \frac{-(2-t)(1-t)l}{4[-2(1-t) + n(2 - 2t + t^2)]^2} < 0.$$

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4 We get that $\frac{\partial w_x^p}{\partial n} = \frac{\partial w_y^p}{\partial n} = \frac{-(2-t)(1-t)l}{4[-2(1-t) + n(2 - 2t + t^2)]^2} < 0.$
\[ q_{pp} = q_{1}^{pp} + \sum_{i=2}^{n} q_{i}^{pp} = \frac{-2n(1-t)-(1-t)^2 + n^2(3-4t+2t^2)}{(n+1)[-4(1-t)+2n(2-2t+t^2)]}. \]

The equilibrium profits of firm 1 and firm \( i, i=2,\ldots,n \), are \( \pi_{1}^{pp} = \left(q_{1}^{pp}\right)^2 - f \) and \( \pi_{i}^{pp} = \left(q_{i}^{pp}\right)^2 \) respectively.

**Proposition 2:** Under patent pool, firm 1 invests in R&D if

\[ k < \left[ \frac{-3(1-s)+n^2(1-s)+n(2-2s+s^2)}{(n+1)[-4(1-s)+2n(2-2s+s^2)]} \right]^2 - \frac{1}{4(n+1)^2} \equiv k_{pp}. \]

**Proof:** Under patent pool, firm 1’s profit under innovation is

\[ \pi_{1}^{pp,rd} = \left[ \frac{-3(1-s)+n^2(1-s)+n(2-2s+s^2)}{(n+1)[-4(1-s)+2n(2-2s+s^2)]} \right]^2 - k, \]

while its profit under no innovation is

\[ \pi_{1}^{pp,nrd} = \frac{1}{4(n+1)^2}. \] Firm 1 innovates if \( \pi_{1}^{pp,rd} > \pi_{1}^{pp,nrd} \), which gives the result. ■

The expression \( k_{pp} \) shows firm 1’s maximum willingness to pay for innovation under patent pool by the input suppliers. The intuition for Proposition 2 is similar to that of Proposition 1.

2.3. **Comparison between non-cooperation and patent pool**

2.3.1. **The effects of a patent pool on the input price and the final goods producers**

The comparison of (6) and (8) gives the following result immediately.

**Proposition 3:** The input prices are higher under non-cooperation to that of under patent pool, irrespective of firm 1’s R&D decision.
Since the inputs are complements, patent pool encourages the input suppliers to reduce respective input prices compared to non-cooperation, since lower input prices increase demands for both inputs, thus increasing the profits of both input suppliers. Hence, patent pool solves the complements problem and reduces the marginal costs of all final goods producers compared to non-cooperation.

Since the patent pool reduces the marginal costs of all final goods producers, it is immediate that if firm 1 does not innovate (i.e., \( t = 1 \)), it increases profits of all final goods producers compared to non-cooperation. However, as we will show below, this may not be the case if firm 1 innovates.

**Proposition 4:** If firm 1 innovates, patent pool reduces (increases) the profit of firm 1 compared to non-cooperation for \( s \in [\frac{2}{n+2}, \frac{n-1}{n}) \) (\( s \in (\frac{n-1}{n}, 1) \)) but it increases the profits of other final goods producers for \( s \in [\frac{2}{n+2}, 1) \).

**Proof:** If firm 1 innovates under non-cooperation and under patent pool, its profit is

\[
\pi_{1nc, rd} = \left[ \frac{-4(1-s) + 2n^2(1-s) + n(2-2s+s^2)}{(n+1)[-6(1-s) + 3n(2-2s+s^2)]} \right] - k \quad \text{under non-cooperation and its profit is}
\]

\[
\pi_{1pp, rd} = \left[ \frac{-3(1-s) + n^2(1-s) + n(2-2s+s^2)}{(n+1)[-4(1-s) + 2n(2-2s+s^2)]} \right] - k \quad \text{under patent pool. We get that}
\]

\[
\pi_{1nc} > (<) \pi_{1pp} \text{ for } s \in [\frac{2}{n+2}, \frac{n-1}{n}) \ (s \in (\frac{n-1}{n}, 1)).
\]

If firm 1 innovates under non-cooperation and under patent pool, the profit of firm \( i, \( i=2,\ldots,n \), is \( \pi_{ic} = \left[ \frac{-2(1-s^2) + n(2-4s+3s^2)}{(n+1)[-6(1-s) + 3n(2-2s+s^2)]} \right]^2 \) under non-cooperation and its profit is
\[ \pi_{ip}^{pp} = \left[ \frac{-2 - s + s^2 + n(2 - 3s + 2s^2)}{(n + 1)[4(1-s) + 2n(2-2s+s^2)]} \right]^2 \] under patent pool. We get that \( \pi_{ip}^{nc} < \pi_{ip}^{pp} \) for \( s \in \left( \frac{2}{n+2}, 1 \right) \).

Although the patent pool reduces the marginal costs of all final goods producers, the marginal cost saving is higher to the non-innovators than to the innovator compared to the situation with non-cooperation. If the technological difference between the innovator and the non-innovators is large (i.e., \( s \in \left( \frac{2}{n+2}, \frac{n-1}{n} \right) \)), the marginal cost saving following the patent pool is significantly higher to the non-innovators compared to the innovator, thus reducing the profit of the innovator under patent pool compared to non-cooperation.

2.3.2. The effects of patent pool on innovation

**Proposition 5:** If \( s \in \left[ \frac{2}{n+2}, s^* \right) \) (\( s \in (s^*, 1) \)), where \( s^* = \frac{3+4n - 7n^2 + \sqrt{9 - 58n^2 + 49n^4}}{4n} \),

patent pool reduces (increases) firm 1’s incentive for innovation compared to non-cooperation among the input suppliers for \( k \in (k^{pp}, k^{nc}) \) (\( k \in (k^{nc}, k^{pp}) \)).

**Proof:** We obtain that \( k^{nc} > (\leq) k^{pp} \) for \( s \in \left[ \frac{2}{n+2}, s^* \right) \) (\( s \in (s^*, 1) \)), where \( s^* = \frac{3+4n - 7n^2 + \sqrt{9 - 58n^2 + 49n^4}}{4n} \).

If \( s \in \left[ \frac{2}{n+2}, s^* \right) \) and \( k^{pp} < k < k^{nc} \), firm 1 innovates only under non-cooperation. In this situation, patent pool reduces firm 1’s incentive for innovation compared to non-cooperation. However, patent pool does not affect firm 1’s incentive for innovation if either
$k < k^{pp} < k^{nc}$ (where firm 1 innovates under patent pool and non-cooperation) or $k^{pp} < k^{nc} < k$ (where firm 1 does not innovate under patent pool and non-cooperation).

If $s \in (s^*,1)$ and $k^{nc} < k < k^{pp}$, firm 1 innovates only under patent pool. In this situation, patent pool increases firm 1’s incentive for innovation compared to non-cooperation. However, patent pool does not affect firm 1’s incentive for innovation if either $k < k^{nc} < k^{pp}$ (where firm 1 innovates under patent pool and non-cooperation) or $k^{nc} < k^{pp} < k$ (where firm 1 does not innovate under patent pool and non-cooperation).

The reason for the above result is as follows. Patent pool (compared to non-cooperation) reduces the marginal costs of the final goods producers under both innovation and no innovation by firm 1. On one hand, the lower marginal cost under patent pool tends to increase firm 1’s incentive for innovation by increasing its profit margin. On the other hand, the lower marginal cost under patent pool makes firm 1 more cost efficient and tends to reduce its incentive for costly innovation. The former effect dominates (is dominated by) the latter if innovation by firm 1 does not reduce (reduces) its input coefficient significantly.

There is another way to explain the above result. Firm 1’s incentive for innovation depends on its difference in profit under innovation and no innovation. On one hand, patent pool (compared to non-cooperation) increases firm 1’s profit under no innovation, thus reducing its incentive for investment in R&D under patent pool. On the other hand, patent pool (compared to non-cooperation) increases (reduces) firm 1’s profit under innovation if its technological improvement through R&D is not large (large), thus increasing (reducing) its incentive for R&D under patent pool. If firm 1’s technological improvement through R&D is large, both the above-mentioned effects tend to reduce firm 1’s incentive for innovation under patent pool compared to non-cooperation. However, if firm 1’s technological improvement
through R&D is small, the above-mentioned second effect dominates the first effect, and patent pool increases firm 1’s incentive for innovation.

2.3.3. The effects of patent pool on the total output

It follows from Propositions 3 and 5 that patent pool has two opposing effects on the total outputs. On one hand, it tends to reduce the input prices, and on the other hand, it may reduce innovation by firm 1. The following proposition shows that, depending on the extent of firm 1’s technological improvement through R&D, patent pool may either increase or decrease the total outputs of firms 1 and 2, thus may have an ambiguous effect on consumer surplus.

**Proposition 6:** Assume that \( s \in \left[ \frac{2}{n+2}, s^* \right) \), where \( s^* = \frac{3 + 4n - 7n^2 + \sqrt{9 - 58n^2 + 49n^4}}{4n} \), and \( k \in (k^{pp}, k^{nc}) \). Patent pool reduces (increases) the total outputs of the final goods producers compared to non-cooperation for \( s \in \left[ \frac{2}{n+2}, s^{**} \right) \) (\( s \in (s^{**}, s^*) \)), where

\[
\frac{2}{n+2} < s^{**} = \frac{-4 + n + 3n^2 - \sqrt{3(-n^2 + n^4)}}{-4 + 3n^2} < s^*.
\]

**Proof:** If \( s \in \left[ \frac{2}{n+2}, s^* \right) \) and \( k \in (k^{pp}, k^{nc}) \), firm 1 innovates only under patent pool. The total outputs of the final goods producers under “non-cooperation with R&D by firm 1” and under “patent pool with no R&D by firm 1” are

\[
q_{nc,rd} = -\frac{2n(1-s) - 2(1-s)^2 + n^2(4 - 6s + 3s^2)}{(n+1)[-6(1-s) + 3n(2 - 2s + s^2)]}
\]

and

\[
q_{pp,rd} = \frac{n}{2(n+1)}
\]

respectively. We get that \( q_{nc,rd} > (<) q_{pp,rd} \) if \( s \in \left[ \frac{2}{n+2}, s^{**} \right) \) (\( s \in (s^{**}, s^*) \)), where

\[
\frac{2}{n+2} < s^{**} = \frac{-4 + n + 3n^2 - \sqrt{3(-n^2 + n^4)}}{-4 + 3n^2} < s^*.
\]
Proposition 6 suggests that although patent pool creates the beneficial input price effect, which solves the complements problem, patent pool’s adverse effect on firm 1’s innovation may dominate its beneficial input price effect, thus reducing the total outputs of the final goods producers under patent pool compared to non-cooperation. Since consumer surplus in our analysis is $q^2/2$, the above result implies that patent pool makes the consumers worse off compared to non-cooperation if the extent of possible technological improvement through firm 1’s R&D is large.

Proposition 5 shows that patent pool increases firm 1’s incentive for innovation for $s \in (s^*,1)$ and $k^{nc} < k < k^{pp}$. In this situation, the total outputs of the final goods producers are lower under “non-cooperation with no R&D by firm 1” to that of under “patent pool with R&D by firm 1”, implying that consumer surplus is higher under patent pool than under non-cooperation. Both the positive input pricing effect and the positive innovation effect help to reduce the marginal costs of final goods production, thus making the consumers better off under patent pool compared to non-cooperation.

We have considered a binary choice for firm 1’s R&D decision. As a result, firm 1 may not innovate under patent pool if the technological improvement through R&D is not very small. However, no innovation by firm 1 under patent pool is an extreme situation and is the artefact of the binary choice. It is now immediate that if firm 1 could choose the extent of technological improvement through R&D, it would always innovate under patent pool but the extent of technological improvement could be lower under patent pool compared to non-cooperation. Hence, even if firm 1’s R&D decision is not a binary choice and firm 1 can choose the extent of technological improvement, the adverse effect of a patent pool on firm 1’s innovation remains, which, in turn, may also make the consumers worse off under patent
pool compared to non-cooperation. The consideration of a binary choice for firm 1’s R&D decision helps us to prove our point in the simplest way.

2.3.4. The effects of patent pool on the profits of the input suppliers

So far we have considered non-cooperation and patent pool by the input suppliers. However, it is important to see whether the input suppliers have the incentive to form the patent pool, and especially when patent pool affects innovation by firm 1.

The profits of the input suppliers are \( \pi_x^{nc} = \pi_y^{nc} = \frac{(n-1+t)^2}{9(n+1)[-2(1-t) + n(2-2t+t^2)]} \) and \( \pi_x^{pp} = \pi_y^{pp} = \frac{(n-1+t)^2}{58(n+1)[-2(1-t) + n(2-2t+t^2)]} \) under non-cooperation and patent pool respectively. It is immediate that if firm 1 either innovates or does not innovate under both non-cooperation and patent pool by the input suppliers, the profits of the input suppliers are higher under patent pool than under non-cooperation.

Now consider the situation where firm 1 innovates under non-cooperation but it does not innovate under patent pool, which occurs for \( s \in \left[ \frac{2}{n+2}, s^* \right] \) and \( k^{pp} < k < k^{nc} \). In this situation, the profits of the input suppliers are \( \pi_x^{nc,rd} = \pi_y^{nc,rd} = \frac{(n-1+s)^2}{9(n+1)[-2(1-s) + n(2-2s+s^2)]} \) and \( \pi_x^{pp,rd} = \pi_y^{pp,rd} = \frac{n}{8(n+1)} \) under non-cooperation and patent pool respectively. Straightforward comparison shows that the profits of the input suppliers are higher under patent pool than under non-cooperation in this situation.

Finally, consider the case where firm 1 innovates under patent pool but it does not innovate under non-cooperation, which occurs for \( s \in (s^*,1) \) and \( k^{nc} < k < k^{pp} \). In this

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5 The intuition follows from Marjit and Mukherjee (2008), which shows in a different context that an input price reduction may either increase or decrease investment in innovation.
situation, the profits of the input suppliers are \( \pi_{x}^{nc,nd} = \pi_{y}^{nc,nd} = \frac{n}{9(n+1)} \) and

\[
\pi_{x}^{pp,rd} = \pi_{y}^{pp,rd} = \frac{(n-1+s)^2}{58(n+1)[-2(1-s) + n(2 - 2s + s^2)]}
\]

under non-cooperation and patent pool respectively. We get that the profits of the input suppliers are higher under patent pool than under non-cooperation in this situation.

The following proposition is immediate from the above discussion.

**Proposition 7:** Patent pool increases the profits of the input suppliers compared to non-cooperation, irrespective of its effect on innovation by firm 1.

### 2.3.5. The effects of patent pool on social welfare

Finally, we want to see the effects of patent pool on social welfare, which is the sum of profits of the input suppliers, the net profits of the final goods producers and consumer surplus. If firm 1 either innovates or does not innovate under both non-cooperation and patent pool, welfare is higher under patent pool compared to non-cooperation. Given the technology level, a lower input price under a patent pool compared to non-cooperation helps to create higher welfare under the former than the latter.

Now consider the case where firm 1 innovates only under non-cooperation, i.e., when \( s \in \left[ \frac{2}{n+2}, s^{*} \right) \) and \( k^{pp} < k^{nc} \). We will see that patent pool may reduce welfare in this situation. Due to the complicated welfare expression, we will consider two numerical examples to show that whether patent pool reduces welfare in this situation depends on the product market competition, given by \( n \).
Assume that \( n=2 \), \( s \in \left[ \frac{1}{2}, s^* = -\frac{17 + \sqrt{561}}{8} \right) \) and \( k^{pp} < k < k^{nc} \). We get in this situation that welfare is higher under “patent pool and no innovation” than under “non-cooperation and innovation” even if we consider maximum welfare under non-cooperation, which occurs at the cost of innovation \( k^{pp} \). However, if we consider that \( n=20 \), \( s \in \left[ \frac{1}{11}, s^* = -\frac{2717 + \sqrt{7816809}}{80} \right) \) and \( k^{pp} < k < k^{nc} \), we get that welfare in this situation is lower (higher) under “patent pool and no innovation” than under “non-cooperation and innovation” for \( s \in \left( \frac{13}{100}, \frac{-2717 + \sqrt{7816809}}{80} \right) \) when we consider minimum welfare under non-cooperation, which occurs at \( k^{nc} \).

The reason for the above result is as follows. If a patent pool reduces innovation, it creates two opposing effects on welfare. On one hand, it tends to increase welfare by reducing the input prices. On the other hand, it tends to reduce welfare by reducing innovation. We have seen that more firms reduce the equilibrium input prices. Since more firms create significantly lower input price under non-cooperation, further benefit from a patent pool due to a lower input price is not significant if the product market competition is significant. Hence, if the number of firms and the technological improvement through innovation are large, the loss from a patent pool due to lower innovation dominates the gain created by the patent pool through lower input prices, thus creating lower welfare under “patent pool and no innovation” compared to “non-cooperation and innovation”.

However, if the product market is very much concentrated (i.e., \( n \) is small), the gain from a patent pool due to a lower input price is significant to outweigh the negative effect of a patent pool on innovation. In this situation, patent pool increases welfare even if it reduces innovation.
Finally, consider the case where firm 1 innovates only under patent pool, which occurs for $s \in (s^*, 1)$ and $k^{nc} < k < k^{pp}$. We get in this situation that welfare is higher under “patent pool and innovation” than under “non-cooperation and no innovation” even if we consider minimum welfare under patent pool, which occurs at the cost of innovation $k^{pp}$. The positive effects of both lower input price and innovation following a patent pool is responsible for this result.

The following result summarises the above discussion.

**Proposition 8:** If patent pool reduces innovation compared to non-cooperation, social welfare may be lower under “patent pool and no innovation” than under “non-cooperation and innovation” if the product market is sufficiently competitive and the technological improvement through R&D is large.

Patent pool solves the complements problem but it may reduce innovation by firm 1. As mentioned above, this trade-off is responsible for the above result. Since the benefit from a patent pool due to a lower input price depends on the number of final goods producers, product market competition plays an important role for the above result.

As already mentioned, no innovation under patent pool is the artefact of the binary choice for firm 1’s R&D decision and we consider this binary choice to prove our point in the simplest way. If we consider a non-binary choice for firm 1’s R&D decision and firm 1 can choose the extent of technological improvement, firm 1 would always innovate under patent pool but the extent of technological improvement chosen by firm 1 could be lower under patent pool compared to non-cooperation. Hence, even if we allow firm 1 to choose the extent of technological improvement, patent pool creates the adverse effect on innovation, which, in effect may reduce social welfare compared to non-cooperation.
3. Conclusion

Many products need to use complementary inputs owned by different firms, which often cooperate or pool their patents. We show the effects of a patent pool on profits, consumer surplus and welfare, highlighting its implications on innovation by a final goods producer.

We show that patent pool may reduce a final goods producer’s incentive for innovation. Hence, although patent pool solves the complements problem, it may have an adverse effect on the technological improvement by the final goods producer. We show that the adverse effect on the technological improvement may dominate the beneficial input price effect of a patent pool, and the patent pool may make the consumers and the society worse off compared to non-cooperation. Thus, in contrast to the existing literature suggesting that a patent pool by the complementary input suppliers makes the consumers and the society better off by solving the complements problem, we show that in a dynamic setup with innovation by the final goods producer, patent pool by complementary input suppliers may make the consumers and the society worse off even if it solves the complements problem. We also show that, although patent pool makes the input suppliers better off, it may reduce the profit of the technologically superior final goods producer compared with non-cooperation among the input suppliers. Hence, the general belief about the positive effects of a patent pool by complementary input suppliers may not hold true under endogenous technology choice.
Appendix

We show in this Appendix that, if the final goods producers differ in terms of their technologies, the input suppliers’ sell inputs to all final goods producers under non-cooperation and patent pool if \( \frac{2}{n+2} \leq s \). If the input suppliers want to charge an input price in a way so that it is not profitable for all firms to purchase inputs at that input price, it is easy to understand that the input suppliers can prevent the non-innovating firms from purchasing inputs but cannot prevent only the innovating firm from purchasing the input. Since the outputs of the innovating firm is always positive whenever the outputs of the non-innovating firms are positive, the input suppliers cannot charge a price that will induce only the non-innovating firms to purchase inputs.

Non-cooperation: First, consider the case of non-cooperation. Consider a symmetric equilibrium for input prices. If the inputs suppliers want to sell the inputs only to the technologically superior final goods producer (i.e., to firm 1, which innovates a new technology and creates technological difference between the final goods producers), the input prices need to be such that it is not profitable for the technologically inferior non-innovating firms to purchase inputs. If the input suppliers supply inputs to firm 1 only, the input demand function is \( q_x = q_y = \frac{s[1-s(w_x + w_y)]}{2} \) and the equilibrium input prices are \( w^m_{x,c,m} = w^m_{y,c,m} = \frac{1}{3s} \).

The outputs of the non-innovating firms are zero at these input prices if \( s \leq \frac{4}{5} \). If \( \frac{4}{5} < s \), the equilibrium input prices need to be \( \hat{w}_x = \hat{w}_y = \frac{1}{2(2-s)} \) to prevent the non-innovating firms from purchasing the inputs. Since \( \hat{w}_x = \hat{w}_y = \frac{1}{2(2-s)} \) is the constrained input price, it is immediate that the input suppliers equilibrium profit from supplying inputs to only firm 1 is
higher if they can charge the input price \( w^{m,m}_x = w^{m,m}_y = \frac{1}{3s} \) than when they need to charge the input price \( \hat{w}_x = \hat{w}_y = \frac{1}{2(2-s)} \).

If the inputs are purchased only by firm 1 at the input prices \( w^{m,m}_x = w^{m,m}_y = \frac{1}{3s} \), which can happen for \( s \leq \frac{4}{5} \), the equilibrium profits of the input suppliers are \( \pi^{m,m}_x = \pi^{m,m}_y = \frac{1}{18} \). We get that, if \( \frac{2}{n+2} \leq s \), \( \pi^{m,m}_x = \pi^{m,m}_y = \frac{1}{18} \) are lower than

\[
\pi^{m}_x = \pi^{m}_y = \frac{(n-1+s)^2}{9(n+1)[-2(1-s) + n(2-2s + s^2)]},
\]

which are the profits of the input suppliers when all final goods producers are supplied the inputs, as considered in the text. Since \( \pi^{m,m}_x = \pi^{m,m}_y = \frac{1}{18} \), i.e., the profits of the input suppliers under the unconstrained input prices \( w^{m,m}_x = w^{m,m}_y = \frac{1}{3s} \), are lower than the input suppliers’ profits from supplying inputs to all final goods producers, it is immediate that if \( \frac{4}{5} < s \) and the inputs suppliers charge

\[
\hat{w}_x = \hat{w}_y = \frac{1}{2(2-s)}
\]

and sell inputs to only firm 1, the input suppliers’ profits are lower from selling the inputs to only firm 1 than from selling the inputs to all final goods producers. This implies that the input suppliers supply inputs to all the final goods producers under non-cooperation for \( \frac{2}{n+2} \leq s \), as considered in the text.

**Patent pool:** Now consider patent pool. If the input suppliers supply inputs to firm 1 only, the input demand function is \( q_s = q_y = \frac{s[1-s(w_x + w_y)]}{2} \) and the equilibrium input prices are
\[ w_{x}^{pp,m} = w_{y}^{pp,m} = \frac{1}{4s}. \] The outputs of the non-innovating firms are zero at these input prices if 
\[ s \leq \frac{2}{3}. \] If \( \frac{2}{3} < s \), the equilibrium input prices need to be \( \hat{w}_{x}^{m} = \hat{w}_{y}^{m} = \frac{1}{2(2-s)} \) to prevent the
non-innovating firms from purchasing the inputs. Since \( \hat{w}_{x} = \hat{w}_{y} = \frac{1}{2(2-s)} \) is the constrained
input price, it is immediate that the input suppliers equilibrium profit from supplying inputs to
only firm 1 is higher if they can charge the input price \( w_{x}^{pp,m} = w_{y}^{pp,m} = \frac{1}{4s} \) than when they
need to charge the input price \( \hat{w}_{x} = \hat{w}_{y} = \frac{1}{2(2-s)} \).

If the inputs are purchased only by firm 1 at the input prices \( w_{x}^{pp,m} = w_{y}^{pp,m} = \frac{1}{4s} \), which
can happen for \( s \leq \frac{2}{3} \), the equilibrium profits of the input suppliers are \( \pi_{s}^{pp,m} = \pi_{y}^{pp,m} = \frac{1}{16} \).
We get that, if \( \frac{2}{n+2} \leq s \), \( \pi_{s}^{pp,m} = \pi_{y}^{pp,m} = \frac{1}{16} \) are lower than
\[ \pi_{s}^{pp} = \pi_{y}^{pp} = \frac{(n-1+s)^2}{58(n+1)[-2(1-s)+n(2-2s+s^2)]}, \] which are the profits of the input suppliers
when all final goods producers are supplied the inputs, as considered in the text. Since
\[ \pi_{s}^{pp,m} = \pi_{y}^{pp,m} = \frac{1}{16}, \] i.e., the profits of the input suppliers under the unconstrained input prices
\[ w_{x}^{pp,m} = w_{y}^{pp,m} = \frac{1}{4s}, \] are lower than the input suppliers’ profits from supplying inputs to all
final gods producers, it is immediate that if \( \frac{2}{3} < s \) and the input suppliers charge
\( \hat{w}_{x} = \hat{w}_{y} = \frac{1}{2(2-s)} \) and sell inputs to only firm 1, the input suppliers’ profits are lower from
selling the inputs to only firm 1 than from selling the inputs to all final goods producers. This
implies that the input suppliers supply inputs to all the final goods producers under a patent pool for \( \frac{2}{n+2} \leq s \), as considered in the text.


