On Stabilisation Policy: Are There Conflicting Implications for Growth and Welfare?

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Abstract

The paper examines the choices for fiscal stabilisation policy that maximise aggregate welfare and long-run growth. This is done in the context of a stochastic dynamic general equilibrium model where premeditated learning provides the engine of human capital accumulation and growth, and technology shocks provide the impulse source of fluctuations. Contrary to existing conventional wisdom, the results indicate a conflict between the two policy objectives: the choice of no stabilisation, associated with maximum growth, is also associated with minimum welfare. Welfare maximisation requires a full stabilisation response to the occurrence of business cycles.

Keywords: Growth; Business cycles; Stabilisation policy

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1 Introduction

Almost every textbook in macroeconomics treats the analysis of short-term movements (i.e., business cycles) and long-term movements (i.e., growth) in economic activity separately. This reflects the, until recently held, conventional wisdom that cyclical fluctuations and economic growth are distinct phenomena – an idea that had been mirrored in the largely separate literatures on short-run cycles and long-run growth. In the Keynesian tradition, the reason for this dichotomy lies in the presumption that cyclical fluctuations arise from short-run market imperfections and rigidities, whereas growth applies to the situation in the long-run when all adjustments have been made and all markets function perfectly. The Real Business Cycles approach endorses this dichotomy as it treats the secular (or growth) component of productivity as an exogenous and deterministic process, unrelated to its stochastic (or cyclical) component which provides the impulse source of fluctuations.

During the late 80’s and early 90’s, some authors proposed models whereby productivity improvements – and sustainable growth – can occur as a consequence of investment decisions by optimising individuals rather than from some exogenously given process (as suggested by the standard neoclassical model). In this line of research, improvements in productivity may be caused by either a learning-by-doing mechanism, whereby investment in physical capital (or any other measure of aggregate economic activity) generates knowledge spillovers that spread over the economy as a whole and improve its efficiency in production (Romer, 1986), by improvements in knowledge that occur as a result of deliberate investment in human capital, like education, training etc., (Lucas, 1988), or by technological changes induced by purposeful activities of entrepreneurs, like R&D, that lead to improvements to the quality or the variety of the reproducible inputs of production (Romer, 1990). In general, models of this class have been categorised in what is now generally known as ‘endogenous growth theory’, due to their property for sustaining
growth in the long-run as a result of *endogenous* decisions by the private sector instead of relying to *exogenous* processes for productivity.

The aforementioned strand of literature threw a new perspective on the possible interactions between growth and cyclical volatility. Soon after its emergence, economists realised the potential of integrating the two approaches under a unified methodological framework as to acquire a better understanding of how these phenomena are related. Stochastic, dynamic general equilibrium models with endogenous processes for either productivity improvements or technological change provide analytical tools through which researchers can illustrate and explain the circumstances under which the variability generated from aggregate fluctuations impinges on trend growth. This is because the engine of growth depends on variables that are affected by the behaviour and actions of individuals. Thus, if macroeconomic volatility affects the magnitude of these variables then it can be linked with long-term growth. Indeed - in recent years – a growing body of theoretical literature has considered this a point worth dwelling on and have pursued its formal analysis by employing models of this vein. In summary, existing analyses show that the effects of different sources of volatility on trend growth – when they are not clear-cut, as in the analyses of Dotsey and Sarte (2000) and Canton (2002) – may depend crucially on preference parameters (e.g., Blackburn and Varvarigos, 2006; de Hek, 1999; Jones *et al.*, 1999; Smith, 1996), technological parameters (e.g., Aghion and Saint-Paul, 1998; Blackburn and Galindez, 2003; Varvarigos, 2006) or the source of volatility itself (Blackburn and Pelloni, 2004; Blackburn and Varvarigos, 2006).\(^1\)

Whatever the underlying mechanism leading to either positive or negative correlation between the two phenomena, the general idea that the growth rate of output can be permanently affected by factors that that are normally deemed relevant only to short-term fluctuations has potentially significant implications for counter-

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\(^1\) A large number of empirical analyses have established a statistically significant relationship between different measures of volatility and average output growth. Indicatively, such evidence can be found on the analyses of Andreou *et al.* (2003), Brunetti (1998), Grier and Perry (2000), Imbs (2002), Kormendi and Meguire (1985), Martin and Rogers (2000) and Ramey and Ramey (1995) among others.
cyclical policies – that is, macroeconomic policies designed to stabilise such fluctuations. It is surprising that, while the potential for different types of policies – either fiscal or monetary – to affect economic outcomes is now well established (both theoretically and empirically), the issue of how counter-cyclical policies may affect such outcomes has so far eluded the attention of most researchers, despite the fact that the smoothing of fluctuations in economic activity is still one of the major goals of government policies. This neglect surely echoes the reluctance of many economists to relate cyclical phenomena with secular trends.2

To the best of my knowledge, the only analyses that have dealt explicitly with this issue are those of Cassou and Lansing (1997), Martin and Rogers (1997), Blackburn (1999), Blackburn and Pelloni (2005). Cassou and Lansing (1997) construct a Romer-type endogenous growth framework (i.e., a model in which physical capital investment generates learning spillovers) to which they incorporate productivity shocks. In this context they conduct different numerical experiments concerning fiscal policy. Specifically, they examine the policy that maximises growth, the policy that maximises welfare and a counter-cyclical policy with the objective of stabilising fluctuations in output. With respect to the later, they argue that it results in improvements in both output growth and aggregate welfare. They also report that, compared with the policy of growth maximisation, there are lower benefits in terms of growth but higher in terms of welfare. Martin and Rogers (1997) present a model in which productivity shocks generate fluctuations in employment. They show that a fiscal policy rule through which labour income is subsidised in bad periods and taxed in good periods can stabilise the economy at its full employment level, achieve maximum average growth and increase welfare. This is because the aggregate level of employment generates learning spillovers that drive human capital accumulation and, consequently, growth in the long-run. Blackburn (1999) considers a stochastic monetary growth model with wage rigidities in which business cycles occur as a result

2 This is somewhat surprising given that some early economists introduced the idea that trend growth is not independent of temporary cycles. Among them, one finds such economists as Joseph Schumpeter, John Hicks and Nicholas Kaldor.
of random monetary and productivity disturbances. The author finds that a counter-
cyclical feedback rule designed to stabilise fluctuations in employment and prices can
actually dampen output growth in the long run. This is because equilibrium
employment is increasing in the variability of the random shocks. As stabilisation
policy reduces the magnitude of this effect it leads to lower trend growth. Blackburn
and Pelloni (2005) modify the analysis of Blackburn (1999) by including physical
capital and assuming that monetary policy reacts systematically to the presence of
exogenous (real and monetary) shocks by altering the money supply in a counter-
cyclical way. They find that the welfare maximising choice of stabilisation policy
parameters is actually the one that maximises trend growth.

All the aforementioned analyses exemplify a message which has been conventional
in existing analyses of growth: given that utility is a monotonically increasing function
of consumption and that (in a dynamic model) consumption can be written in terms
of an initial condition and a growth component, then a policy which is beneficial for
growth, is beneficial for welfare as well. In this paper I consider a situation in which
fiscal policy is utilised as a vehicle for stabilisation. In particular, this policy takes the
form of a feedback rule for the fiscal variable through which the government reacts
to fluctuations in economic activity by decreasing the tax rate during recessions and
increasing it during expansions. One of the novelties of my approach is that the
counter-cyclical stance of fiscal policy is described by a policy parameter which
indicates the degree to which the government intervenes in the economy so as to
stabilise fluctuations in economic activity, as this is captured by the variability of the
technology shocks. In this way I can examine the effects from varying degrees of
stabilisation, treating counter-cyclical policy as a standard parameter. As a result, I am
able to identify analytically the choice of stabilisation that maximises growth and/or
welfare in terms of this single parameter’s values.

The economic environment under which the government exercises stabilisation
policy is one whereby the engine of knowledge improvements and sustainable growth
is the accumulation of human capital through premeditated learning rather than
through serendipitous learning-by-doing. In addition, the impulse source of business cycles comes from technology shocks which are propagated via the private sector’s optimal decisions, causing stochastic fluctuations in labour, human capital and, therefore, growth.

As in other models of the broad area examining the volatility-growth nexus, the equilibrium (sustainable) growth rate is a function of technology shocks. However (in this particular model) the extent to which exogenous shocks cause temporary shifts in the growth rate depends on the magnitude of the government’s stabilisation policy parameter. With this in mind, I examine two scenarios concerning the choice of stabilisation – the counter-cyclical policy that maximises average consumption growth and the counter-cyclical policy that maximises social welfare. Interestingly, my results question previously held ideas concerning stabilisation policy. As it turns out the variability from technology shocks enhances trend consumption growth, hence the growth maximising policy is the one that remains completely unresponsive to the occurrence of temporary disturbances that cause business cycles. Nevertheless, it transpires that the welfare maximising policy is the one that actually eliminates business cycles completely. Surprisingly enough, there is an important part of policy making in which growth and welfare objectives may lie in opposite extremes. The precise mechanism leading to this result reclines with the non-linear manner through which the realisation of random shocks affects economic outcomes, as it will become apparent in the main part of the analysis.

The rest of the paper is organised as follows: Section 2 outlines the basic set-up of the model and Section 3 derives the analytical solution of its dynamic general equilibrium. In Section 4 I describe how policy is designed for counter-cyclical purposes. Section 5 analyses the choice of stabilisation policy that maximises the average growth rate and Section 6 analyses the choice of stabilisation policy that maximises aggregate welfare. In Section 7 I conclude.
2 The Basic Set-up

The economy is populated by a mass of \( N \) immortal, identical agents. To save on notation, I normalise \( N = 1 \). For brevity, I also assume zero population growth. Each agent is both a consumer and a producer of a perishable commodity. Production takes place according to

\[
Y_t = \Phi_t L_t H_t,
\]

where \( Y_t \) denotes output, \( L_t \) is labour effort, \( H_t \) is the existing (previously accumulated) stock of knowledge or human capital, and \( \Phi_t \) is a positively-valued technology shock, identically and independently distributed over time with support on the interval \([\underline{\phi}, \bar{\phi}]\). This random shock has a mean value \( \mu_\phi \), and variance \( \sigma^2_\phi \).

Following the existing literature on models with sustainable growth, I assume that individuals accumulate human capital through deliberate investment in education, training or research (e.g., Lucas, 1988; Razin, 1972; Uzawa, 1965). Agents undertake this investment by combining effort, \( S_t \), together with the existing stock of knowledge, \( H_t \). Therefore, the law of motion for human capital is given by

\[
H_{t+1} = \Omega S_t H_t,
\]

where, for clarity and tractability, I have imposed full depreciation on human capital.

As in each period individuals consume their disposable income, the per-period budget constraint is written as

\[
C_t = (1 - T_t) Y_t,
\]

where \( T_t \in (0,1) \) is a proportional tax rate on income. I assume that the total proceeds from taxation, are used by the government to finance the provision of public goods and services, denoted by \( G_t \), according to a continuously balanced budget.\(^4\)

\(^3\)I choose this type of exogenous shocks as it is associated with the real business cycle approach on economic fluctuations (e.g., Long and Plosser, 1983; Prescott, 1986; Stadler, 1990).

\(^4\)One may think of \( G_t \) as spending on services like law and order, protection of property rights and basic infrastructure. In this particular model (given that it depicts a growing economy) it is assumed
The representative agent receives utility from consumption and disutility from total effort according to

\[ V = E_0 \sum_{t=0}^{\infty} \beta^t \log \left[ C_t - H_t(L_t + S_t)^\epsilon \right], \quad \epsilon > 1, \beta \in (0,1). \tag{4} \]

where \( E_0 \) is the conditional expectations operator and \( \beta \) is the discount factor. This type of utility incorporates the foregone leisure, resulting from the effort devoted to producing output and accumulating human capital, as an input to some type of home production activities: as human capital increases, the efficiency of the individual in such activities increases as well, hence raising disutility because the opportunity cost of total effort (or foregone leisure) becomes higher.\(^5\)

\section{Dynamic General Equilibrium}

This section is devoted to the solution of the model and the derivation of the dynamic, competitive equilibrium which is defined as follows:

**Definition.** Given the initial value \( H_0 > 0 \), a dynamic, competitive equilibrium is a sequence of quantities \( \{C_t, Y_t, L_t, S_t, G_t, T_t, H_{t+1}\}_{t=0}^{\infty} \) such that:

(i) Given \( \{G_t, T_t, \Phi_t\}_{t=0}^{\infty} \), the quantities \( \{C_t, L_t, S_t, H_{t+1}\}_{t=0}^{\infty} \) solve the representative agent’s optimisation problem.

(ii) \( L_t \) and \( S_t \) are stationary.

(iii) The goods market clears every period, i.e., \( Y_t = C_t + G_t \quad \forall t \geq 0 \).

that there is a minimum necessary public spending-to-output ratio, \( \left[ \frac{G_t}{Y_t} \right]_{t=0}^{\text{min}} \). Below this threshold, no productive activity is possible as public spending in such essential services is insufficient to guarantee the abrupt operation of the socio-economic environment. The assumption of a continuously balanced budget for the public sector has been used extensively in the ‘endogenous growth’ literature (e.g., Barro, 1990; Glomm and Ravikumar, 1997; Rebelo, 1991).

\(^5\) This type of preferences has been widely used by various analyses of business cycles and growth (e.g., Blackburn and Varvarigos, 2006; Cassou and Lansing, 1997, 1998; Collard 1999; Hercowitz and Sampson, 1991).
The government’s budget constraint is satisfied every period, i.e., \( G_t = T_t Y_t, \forall t \geq 0 \).

The individual’s objective is to choose sequences for \( \{C_t\}_{t=0}^{\infty}, \{L_t\}_{t=0}^{\infty}, \{S_t\}_{t=0}^{\infty} \) and \( \{H_t\}_{t=0}^{\infty} \) as to maximise (4) subject to sequences for (1), (2), and (3). Denoting the Lagrange multipliers associated with (3) and (2) by \( \lambda_t \) and \( \xi_t \), respectively, the first order conditions associated with the maximisation problem are the following:

\[
1 \frac{1}{C_t - H_t(L_t + S_t)^{\epsilon}} = \lambda_t, \quad (5)
\]

\[
\frac{\epsilon H_t(L_t + S_t)^{\epsilon-1}}{C_t - H_t(L_t + S_t)^{\epsilon}} = \lambda_t(1 - T_t) \Phi_t H_t, \quad (6)
\]

\[
\frac{\epsilon H_t(L_t + S_t)^{\epsilon-1}}{C_t - H_t(L_t + S_t)^{\epsilon}} = \xi_t \Omega H_t, \quad (7)
\]

\[
\xi_t = \beta E_t [\Omega \xi_{t+1} S_{t+1}^{\epsilon} + \beta E_t [\lambda_{t+1}(1 - T_{t+1}) \Phi_{t+1} L_{t+1}]]
- \beta E_t \left[ \frac{(L_{t+1} + S_{t+1})^{\epsilon}}{C_{t+1} - H_{t+1}(L_{t+1} + S_{t+1})^{\epsilon}} \right]. \quad (8)
\]

The first order condition in (5) equates the marginal utility of consumption with the shadow value of wealth. Equations (6) and (7) are the static optimality conditions for the allocation of effort towards working, \( L_t \), and learning, \( S_t \), respectively. The marginal cost of each activity is given by the respective reduction in utility. The marginal benefit from increased employment is given by the utility value of current consumption through the additional output. The marginal benefit from increased learning, as given in (8), includes the expected discounted benefits of additional knowledge that can be gained in the future and the expected discounted utility value of future consumption from the extra future output, minus the discounted expected loss of utility due to foregone leisure (all as a result of the higher human capital stock).

We can begin the solution to the model by multiplying both sides of (8) by \( H_{t+1} \) and substituting (1), (2), (3) and (5) in the resulting expression. It yields
\[ \xi_t H_{t+1} = \beta F_t (\xi_{t+1} H_{t+1}) + \beta \]  

This is a stochastic, difference equation with solution

\[ \xi_t H_{t+1} = \frac{\beta}{1 - \beta}. \]  

(10)

The solution in (10) satisfies the transversality condition for human capital,

\[ \lim_{j \to \infty} \beta^j (\xi_{t+j} H_{t+j+1}) = 0 \]  

and can be verified by direct substitution back in (9).

Notice that after substituting (5), equation (6) can be rewritten as

\[ (L_t + S_t) \gamma = \frac{(1 - T_t) \Phi_t}{\varepsilon} (L_t + S_t). \]  

(11)

Substitution of (1), (2), (3), (10) and (11) in (7) yields (after cancelling out terms from both sides)

\[ \frac{1}{L_t - \frac{L_t + S_t}{\varepsilon}} = \frac{\beta}{(1 - \beta) S_t}. \]  

(12)

Solving this equation for \( S_t \) yields

\[ S_t = Z L_t, \]  

(13)

where \( Z = \frac{(\varepsilon - 1) \beta}{\varepsilon (1 - \beta) + \beta} \).

Now it is straightforward to get the optimal solutions for labour and learning. Substitute (13) in (11), solve for \( L_t \) and then substitute the result back in (13).

Eventually, the optimal solutions turn out to be

\[ L_t = \frac{1}{1 + Z} \left[ \frac{(1 - T_t) \Phi_t}{\varepsilon} \right]^{1/(\varepsilon - 1)}, \]  

(14)

\[ S_t = \frac{Z}{1 + Z} \left[ \frac{(1 - T_t) \Phi_t}{\varepsilon} \right]^{1/(\varepsilon - 1)}. \]  

(15)

**Proposition 1.** A temporary positive (negative) technology shock results in a temporary increase (decrease) of the equilibrium allocation for both labour and learning. A temporary increase (decrease)
in the tax rate results in a temporary decrease (increase) of the equilibrium allocation for both labour and learning.

Proof. Check that $\partial L_t/\partial \Phi_t, \partial S_t/\partial \Phi_t > 0$ and $\partial L_t/\partial T_t, \partial S_t/\partial T_t < 0$ from (14) and (15). ■

In terms of intuition, any event that causes an increase in the marginal benefit from output production (i.e., a temporary increase in $\Phi_t$ and/or a temporary decrease in $T_t$) will induce individuals to act as to increase the marginal cost from producing output, as to restore the original equilibrium. As observed from (11), individuals can achieve this equilibrium adjustment by increasing their effort for either labour or learning. Furthermore, we can see from (13) that in equilibrium any adjustment in one component of total effort has to be followed by an adjustment of the same direction for the other component as well. Accordingly, this leads to the equilibrium observation of Proposition 1.6

4 Stabilisation Policy

The foregoing analysis suggests that the government might be able to use its fiscal policy to counter fluctuations caused by exogenous disturbances with the view to stabilising the economy. It can achieve this by appropriately changing its policy instrument, $T_t$, in response to exogenous shifts in productivity, resulting from the technology shock $\Phi_t$. It is this idea to which I now turn.

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6 The result that learning activities can be pro-cyclical was first obtained by Blackburn and Varvarigos (2006), and it comes in stark contrast with previous conventional wisdom that favours the so-called ‘opportunity cost’ approach (e.g., Dellas and Sakellaris, 2003; Saint-Paul, 1997). According to this view, learning activities should respond counter-cyclically as temporary positive shocks – by increasing the return of output production – increase the opportunity cost of not spending time or effort to directly productive activities. As a result, individuals respond by shifting time or effort towards labour and against activities that bring future rather than current benefits.
I consider the case in which the government implements a programme of state-contingent taxes according to some rule by which $T_t$ responds to fluctuations in economic activity. Naturally, one would imagine that, for the purposes of stabilisation, such a rule would imply relatively high (low) values of $T_t$ during episodes of relatively high (low) activity, which is to say that $T_t$ would respond procyclically. As in other analyses, I assume that policy makers are able to respond to the contemporaneous realisations of exogenous shocks (e.g., Blackburn and Pelloni, 2005; Gali, 1999; Ireland, 1997; Martin and Rogers, 1997). This assumption may be viewed as providing a benchmark scenario in which policy makers are endowed with as much information as possible on which to base their decisions.7

Given the above, I direct my attention to studying the implications of the following feedback policy rule:

$$
T_t = 1 - (1 - \tau) \left( \frac{\mu_\Phi}{\Phi_t} \right)^\zeta.
$$

(16)

This rule has a simple interpretation. We may think that the policy rule incorporates two components: the permanent one is captured by the parameter $\tau \in (0,1)$ while the counter-cyclical part is captured by the parameter $\zeta \in [0,1]$.8 This parameter indicates the extent to which the government engages in stabilisation since, as long as $\zeta \neq 0$, a positive (negative) shock – or, more generally, an expansion (recession) – leads to an

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7 As it happens, the government does, indeed, possess knowledge of the shocks in our model. Even if it cannot observe them directly, it is able to make perfect inferences from other variables about which it has the same information as private agents. For example, observation of $L_t$ from (14), or $S_t$ from (15), would immediately convey the true value of $\Phi_t$.

8 A necessary parameter restriction to ensure that $T_t > 0$ is $1 - \tau < \frac{\varphi}{\mu_\Phi}$ which I assume that holds. I also assume that the minimum public spending-to-output ratio satisfies $\left( \frac{G_t}{Y_t} \right)^{\min} \geq 1 - (1 - \tau) \frac{\mu_\Phi}{\varphi}$. As in Cassou and Lansing (1997), the restriction of balanced budget means that $G_t$ is determined as a residual which absorbs any fluctuations in public revenue resulting from variations in output and tax rates.
increase (decrease) of the tax rate, hence mitigating the original stimulating (adverse) effect on labour and output.

It is straightforward to see how the above policy rule works in stabilising the economy by considering its implications for $L_t$ and $S_t$. Substituting for $T_t$ in (14) and (15) yields

$$L_t = \frac{1}{1+Z} \left[ \frac{(1-\tau)\mu_{\Phi}^{L}}{\varepsilon} \Phi_1^{1-\tau} \right]^{1/(\varepsilon-1)},$$

(17)

$$S_t = \frac{Z}{1+Z} \left[ \frac{(1-\tau)\mu_{\Phi}^{S}}{\varepsilon} \Phi_1^{1-\tau} \right]^{1/(\varepsilon-1)},$$

(18)

The argument for stabilisation policy – and how it is related to economic outcomes – can become transparent if we think of (17) and (18) as implicitly describing a situation where the government follows a state-contingent program of taxes as to stabilise fluctuations in employment, human capital investment and, therefore, output. Obviously, the higher is the chosen value for $\zeta$, the more rigorous is the response policy in the occurrence of cyclical fluctuations. On the one extreme, when $\zeta = 0$ then solutions for labour and learning imply that

$$L_t = \frac{1}{1+Z} \left[ \frac{(1-\tau)\Phi_{\varepsilon}}{\varepsilon} \right]^{1/(\varepsilon-1)},$$

$$S_t = \frac{Z}{1+Z} \left[ \frac{(1-\tau)\Phi_{\varepsilon}}{\varepsilon} \right]^{1/(\varepsilon-1)},$$

hence replicating the outcomes of a scenario whereby labour, human capital, consumption and output incur the full impact of the shocks and the economy fluctuates freely. On the other extreme, when $\zeta = 1$ the solutions for labour and learning imply that

$$L_t = \hat{L} = \frac{1}{1+Z} \left[ \frac{(1-\tau)\mu_{\Phi}}{\varepsilon} \right]^{1/(\varepsilon-1)},$$

9 In another respect, $\zeta$ can be also thought as a policy preference parameter, in the sense that it signifies the willingness of the government to use its policy as to eradicate fluctuations.
meaning that fluctuations in labour, human capital investment and consumption are fully stabilised and the economy as a whole displays the lower possible degree of cyclical activity, caused only by the direct effect of $\Phi_t$ in the production function.

Apparently, the parameter $\tau$ appears in all possible scenarios since it indicates that the government needs to impose taxes in order to raise revenues and finance its spending. My objective is to analyse the implications for both trend growth and aggregate welfare when such a policy is used as a vehicle for stabilisation of economic fluctuations.

## 5 Trend Growth and Stabilisation

Since the growth rate of consumption is equal to $C_{t+1} / C_t = (1 - T_{t+1})\Phi_{t+1}L_{t+1}H_{t+1} / (1 - T_t)\Phi_tL_tH_t$, we can use the results of the previous analysis, as to get

$$\frac{C_{t+1}}{C_t} = \frac{\Omega Z}{1 + Z} \left[ \frac{(1 - \tau) \mu_{\Phi}}{\varepsilon} \right] \frac{\Phi_t^{(1 - \tau)}}{\Phi_t^{1 - \tau}} \times \Phi_t^{(1 - \tau)} \times \Phi_t^{1 - \tau},$$

where $\omega = 1 + 1/\varepsilon > 1$. Obviously, in the absence of shocks (i.e., if $\Phi_t = \mu_0 \forall t$), the economy would move along a path of balanced growth, whereby the growth rate of consumption and output would be equal to the growth rate of human capital – being affected only by the parameters that impinge on the optimal education decisions. However, in the presence of exogenous shocks, the temporary growth rate is responsive to different realisations of the technology shock as these affect output and its components – i.e., labour and human capital. Specifically, the growth rate responds positively at the time- $t + 1$ realisation of the random shock as it has a direct positive effect on output, which is reinforced by the positive effect on labour. The time- $t$ realisation of the technology shock has two conflicting effects on the growth rate. On the one hand, a negative effect arises as a result of the direct
effect on time- \( t \) output, strengthened by the similar response of time- \( t \) labour. On the other had, it has a positive effect via the resulting boost in learning decisions and, therefore, human capital. As it turns out, the first effect dominates and current growth responds negatively on the previous realisation of the shock.

More importantly for my analysis, it is clear from (19) that the growth rate depends on the policy response to the occurrence of exogenous shocks. Clearly, this type of counter-cyclical policy will have implications for the average (or trend) growth rate of consumption. The following Theorem will facilitate the objective of illustrating and explaining these implications.

**Theorem.** Let \( X_1, X_2, \ldots, X_M \) be some statistically independent random variables with means \( \mu_k \) and variances \( \sigma_k^2 \), for \( k = 1, 2, \ldots, M \). Also, let \( F(X_1, X_2, \ldots, X_M) \) be some continuous function. Then \( \text{Mean}[F(\cdot)] \approx J(\sigma_1^2, \sigma_2^2, \ldots, \sigma_M^2) \) such that \( J(\cdot) > 0 \) (\( < 0 \)) \( \iff \) \( \mu_k \) \( > 0 \) (\( < 0 \)) for \( k = 1, 2, \ldots, M \).

**Proof.** Take a second order Taylor series approximation for \( F(\cdot) \) around \( \mu_k \), \( k = 1, \ldots, M \), as to get

\[
F(X_1, \ldots, X_M) \approx F(\mu_1, \ldots, \mu_M) + \sum_{k=1}^{M} F_{X_k}(\mu_1, \ldots, \mu_M)(X_k - \mu_k)
+ \frac{1}{2} \sum_{k=1}^{M} F_{X_k X_k}(\mu_1, \ldots, \mu_M)(X_k - \mu_k)^2.
\]

Taking expectations in both sides and using \( E(X_k) = \mu_k \), \( E(X_k - \mu_k)^2 = \sigma_k^2 \) for \( k = 1, 2, \ldots, M \), yields

\[
\text{Mean}[F(X_1, \ldots, X_M)] \approx F(\mu_1, \ldots, \mu_M) + \frac{1}{2} \sum_{k=1}^{M} F_{X_k X_k}(\mu_1, \ldots, \mu_M)\sigma_k^2. \quad \blacksquare
\]

Now, we are able to analyse the effects of stabilisation policy on long-run (trend) growth.
Proposition 2. The growth maximising counter-cyclical policy is the least rigorous possible in terms of stabilisation (i.e., $\zeta = 0$).

Proof. Apply the Theorem to equation (19) as to get

$$\text{Mean} \left( \frac{C_{t+1}}{C_t} \right) \approx \frac{\Omega Z \left[ (1 - \tau) \mu_\Phi \right]}{1 + Z \varepsilon} \left[ 1 + \frac{\sigma^2}{2 \mu_\Phi^2} (\omega (1 - \zeta) [\omega (1 - \zeta - 1) + (\zeta - 1)(\zeta - 2)] + \omega(\omega - 1) + 2 \right].$$

Given the above we can find $\partial \text{Mean}(C_{t+1} / C_t) / \partial \zeta = -2 \omega^2 (1 - \zeta) - 3 + 2 \zeta + \omega$ and $\partial^2 \text{Mean}(C_{t+1} / C_t) / \partial \zeta^2 = 2(\omega^2 + 1) > 0$. Now observe that for $\zeta = 0$,

$$\text{Mean} \left( \frac{C_{t+1}}{C_t} \right)_{\zeta=0} \approx \frac{\Omega Z}{1 + Z \varepsilon} \left[ 1 + \frac{\sigma^2}{2 \mu_\Phi^2} \omega(\omega - 1) + 2 \right]$$

while for $\zeta = 1$,

$$\text{Mean} \left( \frac{C_{t+1}}{C_t} \right)_{\zeta=1} \approx \frac{\Omega Z}{1 + Z \varepsilon} \left[ (1 - \tau) \mu_\Phi \right] \left[ 1 + \frac{\sigma^2}{2 \mu_\Phi^2} \right]$$

therefore $\text{Mean} \left( \frac{C_{t+1}}{C_t} \right)_{\zeta=0} > \text{Mean} \left( \frac{C_{t+1}}{C_t} \right)_{\zeta=1}$. The foregoing analyses reveals that average consumption growth attains a maximum at the corner solution $\zeta = 0$. ■

The intuition behind this result is the following: inspection of (19) reveals that the growth rate of consumption is convex in both realisations of the technology shock, as long as $\zeta = 0$. This demonstrates the fact that the gain (loss) in growth as a result of a temporary increase in $\Phi_{t+1}$ ($\Phi_t$) is more (less) pronounced than the loss (gain) in growth generated by a decrease in $\Phi_{t+1}$ ($\Phi_t$) of equal magnitude. As a result, the volatility generated from the technology shocks is beneficial for the average growth rate – something evident from the fact that for $\zeta = 0$, the coefficient on $\sigma^2_\Phi$ is positive. Consequently, a policy targeting at growth maximisation, will allow the full impact incurred from cyclical volatility and will not act as to eliminate it.
Note that, qualitatively, this result is identical if instead of using the growth rate of consumption we use the growth rate of output. To see this, use (2), (17) and (18) in (1) as to get

\[
\frac{Y_{t+1}}{Y_t} = \frac{\Omega Z}{1 + Z} \left[ \frac{(1-\tau)\mu^\zeta}{\epsilon} \right]^{\omega - 1} \Phi_t \Omega^{(\omega-1)(1-\zeta)+1} \Phi_{t+1}.
\]  

(20)

From (20), it is clear that even the most rigorous stabilisation policy (i.e., \(\zeta = 1\)) will not be able to eliminate the variability of output growth completely, as it cannot eradicate the direct effect of the shocks on output growth. Even so, employing a procedure similar to the previous one, we can find that the long-run output growth rate - i.e., \(\text{Mean}(Y_{t+1}/Y_t)\) - is equal to

\[
\frac{\Omega Z}{1 + Z} \left[ \frac{(1-\tau)\mu^\zeta}{\epsilon} \right]^{\omega - 1} \left[ 1 + \frac{\sigma^2}{2\mu^\zeta} \{2 + [(\omega - 1)(1 - \zeta) + 1]((\omega - 1)(1 - \zeta)) \} \right],
\]

which is clearly monotonically decreasing in \(\zeta\). As a result, a policy with the objective of maximising long-term output growth should remain completely unresponsive to the occurrence of exogenous shocks.

6 Stabilisation and Welfare

Having analysed the long-term growth implications arising from stabilisation policy, I now proceed to illustrate and analyse these implications in terms of social welfare.

After substitution of (1), (3), (11) and (12), the term \(C_t - H_t(L_t + S_t)^\zeta\) can be reduced to the expression

\[
\frac{1 - \beta}{\beta} (1 - T_t) \Phi_t S_t H_t.
\]

(21)

Next, notice that recursive substitution in (2) leads to

\[
H_t = \Omega^t S_{t-1} S_{t-2} \cdots S_0 H_0.
\]

(22)

Using (16) and (22) in (21) yields
\[ \Omega' \frac{1-\beta}{\beta} H_0 (1-\tau) \mu_\phi \Phi_\phi^{-\varepsilon} S_t S_{t-1} S_{t-2} \cdots S_0, \quad (23) \]

therefore, the term \( \log \left[ C_t - H_t (L_t + S_t)^\varepsilon \right] \) is equal to

\[
\log \left[ \Omega' \frac{1-\beta}{\beta} H_0 (1-\tau) \right] + \zeta \log (\mu_\phi) + (1-\zeta) \log (\Phi_\phi).
\]

\[ + \log (S_t) + \log (S_{t-1}) + \ldots + \log (S_0) \quad (24) \]

Using (18), we observe that

\[
\log (S_t) = \log \left[ \frac{Z}{(1+Z)e^{\omega-1}} \right] + (\omega-1) [\log(1-\tau) + \zeta \log (\mu_\phi) + (1-\zeta) \log (\Phi_\phi)]. \quad (25)
\]

Substitution of (25) for \( S_t, S_{t-1}, S_{t-2}, \ldots \) in (24) and some straightforward algebra yields

\[
V = \sum_{t=0}^{\infty} \beta^t \left[ \Lambda_t + \varrho_t \zeta \log (\mu_\phi) + (1-\zeta) \log (\Phi_\phi) \right], \quad (26)
\]

where \( \Lambda_t = \log \left[ \Omega' \frac{1-\beta}{\beta} H_0 \right] + \varrho_t \log (1-\tau) + (t+1) \log \left[ \frac{Z}{(1+Z)e^{\omega-1}} \right] \)

and \( \varrho_t = 1 + (\omega-1)(t+1) \).

The above expression for expected lifetime utility, can be utilised for the derivation of the welfare implications of stabilisation policy.

**Proposition 3.** The welfare maximising counter-cyclical policy is the most rigorous possible in terms of stabilisation (i.e., \( \zeta = 1 \)).

**Proof.** Application of the Theorem in (26) yields (after some algebraic manipulation)

\[
V \approx \sum_{t=0}^{\infty} \beta^t \left[ \Lambda_t + \varrho_t \log (\mu_\phi) - \varrho_t (1-\zeta) \frac{\sigma_\phi^2}{2 \mu_\phi^2} \right].
\]

Obviously, the above expression implies that \( \partial V / \partial \zeta > 0 \), therefore it is maximised at the highest value for the stabilisation policy parameter, which is \( \zeta = 1 \). ■
The foregoing analysis reveals that, for an important part of macroeconomic policy making, an interesting outcome may arise. This outcome is associated with the scenario whereby the policy objectives for growth and welfare can diverge. Indeed, in this particular model, a more rigorous counter-cyclical reaction is beneficial for welfare while, at the same time, it may correspond to a reduction in trend growth. More strikingly, the policy choice of no stabilisation which maximises long-run growth, is the one that is associated with the lowest level of social welfare. Although counter-intuitive at first glance, the aforementioned result can be logically explained once we understand the precise nature through which stabilisation policy works.

Stabilisation policy has the objective of minimising the variability imposed by various exogenous disturbances and propagated into aggregate economic activity. As we have already seen, to understand the effects of variability on long-term growth specific attention must be given, not only to the way through which different shock realisations affect various economic outcomes temporarily, but also to the non-linear manner of these impacts. In more conventional deterministic models, the growth and welfare objectives of various policies are largely coincident simply because any policy that is beneficial for long-run growth enhances welfare as well. This occurs because social welfare is a monotonically increasing function of the growth component of aggregate consumption which, along a balanced growth path, corresponds to the rate of change of output. The stochastic model presented here generates a different mechanism: the fact that growth enhances welfare in a monotonous way does not necessarily imply that the non-linearity with respect to the various shocks must coincide as well. As it turns out, in my particular model, the growth rate is convex to the realisations of productivity shocks but welfare is, actually, concave. Hence, a policy of stabilisation that achieves maximum benefits in terms of welfare results in loses in terms of growth – both effects generated by the reduction in cyclical volatility.
7 Conclusions

In this paper I have employed a simple, stochastic dynamic general equilibrium model of cycles and growth, and utilised it as to provide analytical results concerning the different degrees of stabilisation policy associated with maximisation of both aggregate welfare and long-run growth. The analysis yielded an interesting conflict between these two objectives: if the government follows a growth-maximising objective, remaining completely unresponsive to the occurrence of cyclical volatility, then such a policy will lead to minimum welfare. As welfare becomes progressively larger following a strengthened counter-cyclical stance of policy makers, its maximisation actually requires the strongest possible stabilisation policy, even if such an action necessitates a reduction in consumption growth.

One virtue of the model presented here is its tractability. Through this, the analysis benefits from analytical results, detail of all the mechanisms involved and clarity of intuition. However, such tractability has to come at a cost. In this case, the cost takes the form of restrictions I had to impose as to ensure the derivation of closed-form solutions. Since this is an analysis in which the specific characteristics of the business cycle may have a prominent role, the most important restriction is that I have focused exclusively on the amplitude of the cycle, while I absconded from its persistence. It is true that the observed persistence of business cycles is an important stylised fact and, as Fatas (2000) has already shown, it may provide additional insights on the interrelation between business cycles and economic growth. Naturally, one would expect that such additional insights will provide ample scope for extending the analysis of the important implications generated by policies designed to mitigate fluctuations. This is certainly an issue worth pursuing through further research.
References


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