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Inflated Ordered Outcomes

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Abstract

We extend Harris and Zhao (2007) by proposing a (Panel) Inflated Ordered Probit model, and demonstrate its usefulness by applying it to Bank of England Monetary Policy Committee voting data.

Keywords: Panel Inflated Ordered Probit, random effects, inflated outcomes, voting, Monetary Policy Committee.

JEL Classification: E5, C3.

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1 Introduction

In a recent paper, Harris and Zhao (2007) propose a Zero-Inflated Ordered Probit (ZIOP) model to take into account an ‘inflated’ number of observations at one end of an ordered choice spectrum. However, in other applied situations, an excess number of observations may occur elsewhere. For instance, Fabiani et al. (2006) find that European manufacturing firms overwhelmingly leave prices unchanged rather than raise or lower them; similarly, Harris and Spencer (2009) find that Bank of England Monetary Policy Committee (MPC) members vote significantly more often to leave interest rates unchanged instead of reducing or increasing them. In both cases the choices faced by firms and MPC members can be construed as having a clear implicit ordering: reduce, leave unchanged, increase, such that the middle outcome, and not an outcome at one end of the choice spectrum, is ‘inflated’. In light of these examples, we propose an econometric model that has applications in panel or cross-sectional settings, where the choice spectrum is dominated by one particular outcome, irrespective of its position in the ordering. Our model is applied to policy rate votes cast by Bank of England MPC members, and captures voting behavior well.

2 Model

Assume that the choice set is characterized by three ordered outcomes - 0, 1, 2 - where the middle outcome represents the ‘inflated’ category. Generalizations to situations with more outcomes and/or inflation in other categories are obvious. The formal starting point is an underlying latent variable, $q_{it}$ for unit $i$, in period $t$, which is a linear in parameters ($\beta$) function of a vector of observed characteristics $x_{it}$ and a random error $u_{it}$. This latent binary variable represents an overall propensity to choose the inflated category over any other,

$$q_{it}^* = x_{it}' \beta + u_{it}. \quad (1)$$

This represents a panel indexed model; $t$ would be omitted for a cross-sectional one. Label expression (1) the splitting equation (SE). (1) states that unit $i$ has an overall augmented propensity to choose the inflated outcome over all others. We propose a two-regime scenario such that for units in regime $q_{it} = 0$, the inflated outcome is observed; but for those in $q_{it} = 1$ we may witness any of the possible outcomes in the choice set $\{0, 1, 2\}$. Membership of either regime ($q_{it} = 0, q_{it} = 1$) is never observed and is identified by data.

For units in regime $q_{it} = 1$, an underlying latent variable $y_{it}^*$ is specified as a linear in

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1 The term ‘inflated’ is not related to the phenomenon of rising prices, but the fact that the overwhelming majority of empirical observations fall into one particular choice category, which hence appears ‘inflated’. 

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parameters function of a vector of observed characteristics $z_{it}$, with unknown weights $\gamma$ and a random disturbance term $\varepsilon_{it}$,

$$y_{it}^* = z_{it}'\gamma + \varepsilon_{it}. \quad (2)$$

For these units, outcome probabilities are determined by an ordered probit (OP) model. Assuming joint normality of $\varepsilon$ and $u$, final outcome probabilities are given by

$$\Pr(y_{it} = 0 | x_{it}, z_{it}) = \Phi_2(x_{it}' \beta, -z_{it}' \gamma; -\rho_{eu})$$

$$\Pr(y_{it} = 1 | x_{it}, z_{it}) = [1 - \Phi(x_{it}' \beta)] + \{\Phi_2(x_{it}' \beta, \mu - z_{it}' \gamma; -\rho_{eu}) - \Phi_2(x_{it}' \beta, -z_{it}' \gamma; -\rho_{eu})\}$$

$$\Pr(y_{it} = 2 | x_{it}, z_{it}) = \Phi_2(x_{it}' \beta, z_{it}' \gamma - \mu; \rho_{eu}) \quad (3)$$

where $\Phi_2(\cdot; \rho_{eu})$ denotes the cumulative distribution function of the standardized bivariate normal distribution with correlation coefficient $\rho_{eu}$, and $\mu$ represents the usual OP boundary parameter(s). In this way, the probability of outcome 1 has been ‘inflated’: to observe $y_{it} = 1$ requires either that $q_{it} = 0$; or jointly that $q_{it} = 1$ and that $0 < y_{it}^* \leq \mu$. Observationally equivalent category 1 outcomes, can hence arise from two distinct sources. We term this new econometric model a (Panel) Inflated Ordered Probit (PIOP) model. The estimated probabilities in equation (3) are obtained by maximizing the log-likelihood function $L(\theta)$ with respect to the parameter vector $\theta = (\beta', \gamma', \mu, \rho_{eu})'$. For the three-outcome case with each unit $i$ observed for $t = 1, \ldots, T_i$ time periods, $L(\theta) = \sum_{i=1}^{N} \sum_{t=1}^{T_i} \sum_{j=0}^{2} d_{ijt} \ln [\Pr(y_{it} = j | x_{it}, z_{it})]$ where $d_{ijt}$ is an indicator function.

In our application we have panel data, and we include random unobserved effects - $\alpha_i$ and $e_i$ - in equations (1) and (2), such that

$$q_{it}^* = x_{it}' \beta + \alpha_i + u_{it} \quad \text{and} \quad y_{it}^* = z_{it}' \gamma + e_i + \varepsilon_{it}. \quad (4)$$

Unlike the usual panel data setting there is no concern of these unobserved effects being correlated with covariates, as they are not individual specific. However, this innovation complicates estimation and means that each unit’s $it$ likelihood contributions are no longer independent, and the likelihood for each $i$ is the product over $T_i$. These unobserved effects also need to be integrated out of the likelihood function. We undertake this using simulation techniques on the (usual) assumption that $\alpha_i \sim N(0, \sigma_{\alpha}^2)$ and $e_i \sim N(0, \sigma_{e}^2)$, using Halton sequences of length 500.
3 Application

We apply the model to votes cast by Bank of England MPC members at monthly meetings for the period August 1997 to December 2011 during which two notable empirical regularities characterized members’ votes. First, even with the arrival of new economic information, members voted to adjust the policy rate infrequently, leading to an observable ‘excess’ of votes to leave it unchanged. Figure 1 plots members’ vote distribution where the proportion of no-change votes is markedly larger than votes to raise or decrease the policy rate. Second, votes to change the policy rate overwhelmingly occur in 25 basis point increments, making the data well suited for a discrete choice approach. Observed votes are thus classified into three categories: \( y_{it} = 0 \) (rate reduction); \( y_{it} = 1 \) (no-change); and \( y_{it} = 2 \) (rate increase) such that the OP component captures votes to lower, leave unchanged, or raise the policy rate, and the SE captures the propensity to change or not change \( \text{per se} \). No-change decisions can hence arise from each equation.

OP equation votes are modeled as a function of the Bank’s quarterly modal projections for inflation and output growth at the eight and four quarter horizons, respectively, modified as in Goodhart (2005), and expressed in terms of the deviation from the inflation target and an assumed 2.4% rate of potential output growth. We denote these variables \( \pi_{\text{Dev},t} \) and \( \text{GAP}_t \), respectively. The SE models the effects of various forms of uncertainty associated with voting decisions. We include (i) the uncertainty parameter associated with the MPC’s inflation forecast at the eight quarter horizon (\( \sigma \)), proposing that higher values are associated with a greater reluctance to change rates, (ii) an Inflation Report dummy (IR), where a one denotes an Inflation Report release month (February, May, August, November), and zero otherwise; we posit that members are more likely to change in these months, as they are better informed about future economic conditions, and (iii) the volatility of the FTSE 100 index for all trading days between scheduled MPC meetings (FTSE), to capture financial

\[^2\text{The government set inflation target changed in January 2004 from 2.5\% RPIX (Retail Price Index, excluding mortgage interest payments) inflation to 2\% CPI (Consumer Price Index) inflation.}\]

\[^3\text{Our data also encompasses the effective zero lower-bound (ZLB) period from June 2009 onwards. As this period effectively reduces members’ choices to no-change and increase, for this period the option of a rate reduction vote is removed. Due to the reduced sample size in this period, all relevant parameters are restricted to be the same both pre- and post-June 2009. However, our estimation results did not change significantly when the ZLB period was excluded.}\]

\[^4\text{For full details of this construction method see Besley et al. (2008) and Harris and Spencer (2009).}\]

\[^5\text{See http://www.bankofengland.co.uk/publications/Pages/inflationreport/irprobab.aspx.}\]

\[^6\text{Aoki (2003) finds that as the amount of data uncertainty associated with a variable increases, its information content should be discounted. This causes a dampening of the response coefficients in an optimal policy rule, as a result of the policymaker proceeding more cautiously. Whilst ours is not a theoretical contribution, we apply similar intuition.}\]

\[^7\text{Revisions to the MPC’s forecasts in non-Inflation Report months provide no substitute for the “complete reassessment of all the evidence that is involved in a full forecasting round” (Budd (1998), p.1790).}\]
market uncertainty. Finally there is evidence that internal and external MPC members exhibit different voting patterns at meetings (Gerlach-Kristen, 2009). We thus include a dummy variable (TYPE) with value one denoting an external member and zero otherwise in both equations: in the SE it gauges the extent to which these groups differ in their propensities to change per se; in the OP equation it captures the extent to which members differ in their propensities to tighten rates relative to loosening them.

Three PIOP variants were estimated: a ‘baseline’ PIOP model without correlated errors or random unobserved effects (REs); a second model with correlation between the errors of the OP and SE components (PIOPCOR); and a model with correlated errors and REs in each of the SE and OP equations (PIOPCRE). Results are presented in Table 1. Other than TYPE, whose significance diminishes with the introduction of correlated errors and REs, all variable estimates are significant and robust to these innovations. However, with all our information criteria (AIC, BIC, CAIC) measures identifying PIOPCRE as the preferred specification, we focus on these results, which are characterized by highly significant random effects, denoted \( \sigma^2_{e} \) and \( \sigma^2_{\mu} \), and correlated errors, \( \rho_{eu} \). In the OP equation, \( \pi_{Dev} \) and \( \text{GAP}_t \) are statistically significant and correctly signed: positive (negative) values of these variables increase the probability of a vote for a rate hike (cut). In the SE, the sign on \( \text{IR} \) indicates that during Inflation Report months, members are more likely to vote to change rates. \( \text{FTSE} \) is positively signed, indicating that changes in interest rates are an increasing function of financial market volatility. Finally, \( \pi_{\sigma} \) has the effect of reducing the propensity to change rates. To demonstrate the model’s usefulness, Figure 2 illustrates the effect of different \( \pi_{\sigma} \) values on the overall probabilities to change rates, holding other variables at: \( \pi_{Dev} = 0.35; \text{GAP}_t = 0.1; \text{TYPE}_\text{OP} = \text{TYPE}_\text{SE} = 0; \text{FTSE} = 1.8; \) and \( \text{IR} = 1 \). For lower values of \( \pi_{\sigma} \) members still vote to raise rates, attributable to the fact that the economy is ‘overheating’ \( (\pi_{Dev} > 0, \text{GAP}_t > 0) \); however, as \( \pi_{\sigma} \) increases, the probability of a rate hike falls, such that for \( \pi_{\sigma} \geq 0.89 \), a vote to leave rates unchanged is predicted. Thus even if the Bank forecasts inflation to be above target - which has the effect of increasing the probability of a vote to increase rates - a vote to leave the policy rate unchanged might still be expected if a high degree of uncertainty surrounds the inflation forecast itself. Moreover, for \( \pi_{\sigma} \geq 0.89 \) we have

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8 All model variants outperformed the basic ordered probit model in terms of AIC, BIC and CAIC.

9 Here, unobserved individual heterogeneity may be capturing the time-invariant effect of TYPE. Marginal effects (not reported due to space constraints) available on request.

10 Similar findings were found for other constructed measures of financial market uncertainty, including ‘TED-spread’ type measures and overnight rate volatility. This finding is deserving of further investigation.

11 The probability of voting for a rate reduction is not plotted, as in this example the probability of reducing rates remains close to zero.
that \( \Pr (y_{it} = 1) = \max \{ \Pr (y_{it} = 0), \Pr (y_{it} = 1), \Pr (y_{it} = 2) \} \), such that in expression (3) 

\[
[1 - \Phi (x_{it}' \beta)] > \{ \Phi_2 (x_{it}' \beta, \mu - z_{it}' \gamma; -\rho_{\text{eu}}) - \Phi_2 (x_{it}' \beta, -z_{it}' \gamma; -\rho_{\text{eu}}) \}.
\]

(5)

The above decomposition of the overall decision not to change into its SE and OP components is shown in Figure 2 using dotted lines: for higher \( \pi_\sigma \) values the overall probability of no change is increasingly dominated by the SE. Under such conditions, members may be construed as opting to ‘wait and see’, voting to change rates when there is more certainty surrounding the inflation forecast itself. The PIOP model provides a way of modeling such effects.

### 4 Conclusion

This paper has extended Harris and Zhao (2007) in a number of directions through proposing a PIOP model. In our application the model performs well; results suggest the presence of significant correlation between the SE and OP equations, and also of member-specific unobserved effects in the two underlying latent equations. Most significantly, we have provided a possible explanation for why MPC members’ votes are dominated by decisions to leave the policy rate unchanged; specifically, accounting for such observational equivalence is achieved through the introduction of two distinct data generating processes. We envisage that such a generalization to the ZIOP model will be useful in a wide range of applications.

### References


Figure 1: MPC Members’ Vote Distribution, August 1997 - December 2011
Figure 2: Overall probability of voting to raise rates, no change, and its decomposition as a function of increasing forecast uncertainty ($\pi_\sigma$)
## Table 1: Estimation Results

<table>
<thead>
<tr>
<th>OP Equation</th>
<th>PIOP</th>
<th>PIOP_COR</th>
<th>PIOP_CRE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.777 (8.463)**</td>
<td>0.047 (0.321)**</td>
<td>0.386 (0.885)</td>
</tr>
<tr>
<td>GAP_t</td>
<td>0.299 (3.566)**</td>
<td>0.281 (4.148)**</td>
<td>0.397 (3.745)**</td>
</tr>
<tr>
<td>(\pi_{\text{Dev},t})</td>
<td>0.609 (12.76)**</td>
<td>0.516 (15.06)**</td>
<td>0.597 (8.323)**</td>
</tr>
<tr>
<td>TYPE_OP</td>
<td>-0.340 (3.616)**</td>
<td>-0.099 (1.083)</td>
<td>-0.087 (0.154)</td>
</tr>
<tr>
<td>(\mu)</td>
<td>1.753 (13.51)**</td>
<td>1.428 (9.84)**</td>
<td>1.869 (10.02)**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Splitting Equation (SE)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.4079 (1.389)</td>
<td>0.706 (2.642)**</td>
<td>1.434 (1.631)</td>
</tr>
<tr>
<td>(\pi_{\sigma})</td>
<td>-0.1512 (5.556)**</td>
<td>-0.180 (7.361)**</td>
<td>-0.247 (5.459)**</td>
</tr>
<tr>
<td>IR</td>
<td>0.6895 (5.462)**</td>
<td>0.352 (3.561)**</td>
<td>0.406 (1.663)*</td>
</tr>
<tr>
<td>FTSE</td>
<td>0.3814 (6.553)**</td>
<td>0.416 (6.945)**</td>
<td>0.462 (4.118)**</td>
</tr>
<tr>
<td>TYPE_SE</td>
<td>0.4870 (4.207)**</td>
<td>0.470 (4.358)**</td>
<td>0.343 (0.765)</td>
</tr>
<tr>
<td>(\rho_{\text{SE}})</td>
<td>-</td>
<td>0.902 (13.20)**</td>
<td>0.8918 (7.363)**</td>
</tr>
<tr>
<td>(\sigma_{\sigma}^2)</td>
<td>-</td>
<td>-</td>
<td>0.550 (3.086)**</td>
</tr>
<tr>
<td>(\sigma_{\alpha}^2)</td>
<td>-</td>
<td>-</td>
<td>0.730 (4.801)**</td>
</tr>
</tbody>
</table>

### Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>AIC</th>
<th>BIC</th>
<th>CAIC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1933.8586</td>
<td>1908.9307</td>
<td>1800.6215</td>
</tr>
<tr>
<td></td>
<td>1997.2410</td>
<td>1978.6513</td>
<td>1883.0186</td>
</tr>
<tr>
<td></td>
<td>-961.9293</td>
<td>-948.9653</td>
<td>-893.8108</td>
</tr>
</tbody>
</table>

### Sample Proportions and Average Predicted Probabilities

<table>
<thead>
<tr>
<th></th>
<th>Down</th>
<th>No change</th>
<th>Up</th>
</tr>
</thead>
<tbody>
<tr>
<td>Down</td>
<td>0.1873 [0.2026]</td>
<td>0.1873 [0.1965]</td>
<td>0.1873 [0.1828]</td>
</tr>
<tr>
<td>No change</td>
<td>0.6827 [0.6724]</td>
<td>0.6827 [0.6766]</td>
<td>0.6827 [0.6864]</td>
</tr>
<tr>
<td>Up</td>
<td>0.1300 [0.1250]</td>
<td>0.1300 [0.1269]</td>
<td>0.1300 [0.1307]</td>
</tr>
</tbody>
</table>

\(z\)-statistics in parentheses (·). Number of obs. = 1538.

***/**/Denotes two-tailed significance at one / five / ten percent levels.

* Average predicted probabilities in square [·] brackets.

\(\dagger\) Akaike Information Criterion; \(\ddagger\) Bayesian Information Criterion;

\(\ddagger\ddagger\) Consistent Akaike Information Criterion.