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This chapter describes the process of building a mathematical model using rigid bodies and elastic structures to represent body segments and various ways of representing the force generating capabilities of muscle. Direct and indirect methods of determining the physical parameters associated with these elements are described. Before using a model to answer a research question it is first necessary to establish that the model is an adequate representation of the real physical system. This process of model evaluation by comparing model output with real data is discussed. Examples of applications of both forward dynamics and inverse dynamics computer modelling are given.

Computer Simulation Modelling in Sport

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Introduction

Experimental Science aims to answer research questions by investigating the relationships between variables using quantitative data obtained in an experiment and assessing the significance of the results statistically (Yeadon and Challis, 1994). In an ideal experiment the effects of changing just one variable are determined. While it may be possible to change just one variable in a carefully controlled laboratory experiment in the natural sciences, the situation is problematic in the sports sciences in general and in sports biomechanics in particular. If a typical performance in a sport such as high jumping is to be investigated then any intervention must be minimal lest it make the performance untypical. For example if the intent is to investigate the effect of run up speed on the height reached by the mass centre in flight, asking the jumper to use various lengths of run up might be expected to influence jumping technique minimally if the athlete normally does this in training. In such a situation the run up speed may be expected to vary as intended but other aspects of technique may also change. Faster approaches may be associated with a greater stride length and a more horizontal planting of the takeoff leg. As a consequence the effects of a faster approach may be confounded by the effects of larger plant angles and changes in other technique variables. In order to isolate the relationship between approach speed and jump height statistical methods of analysis that remove the effects of other variables are needed (e.g. Greig and Yeadon, 2000). For this to be successful there must be a sufficient quantity and range of data to cope with the effects of a number of variables.

Theoretical approaches to answering a research question typically employ a model that gives a simplified representation of the physical system under study. The main advantage of such a model is that ideal experiments can be carried out since it is possible to change just one variable. This

chapter will describe theoretical models used in sports biomechanics, detailing their various components and discussing their strengths and weaknesses.

Models may be used to address the Forward Dynamics problem and the Inverse Dynamics problem. In the Forward Dynamics problem the driving forces are specified and the problem is to determine the resulting motion. In the Inverse Dynamics problem the motion is specified and the problem is to determine the driving forces that produced the motion (Zatsiorsky, 2002). Both of these types of problem will be addressed using various modelling approaches and their relative advantages will be discussed.

This chapter will first describe the process of building a mathematical model using rigid bodies and elastic structures to represent body segments and various ways of representing the force generating capabilities of muscle. Direct and indirect methods of determining the physical parameters associated with these elements will be described. Before using a model to answer a research question it is first necessary to establish that the model is an adequate representation of the real physical system. This process of model evaluation by comparing model output with real data will be discussed. Examples of applications of computer modelling will be given along with guidelines on conducting a study and reporting it.

The Forward Dynamics Problem

In the Forward Dynamics problem the driving forces are specified and the problem is to determine the resulting motion. Muscle forces or joint torques may be used as the drivers in which case the joint angle time histories will be part of the resulting motion. If joint angle time histories are used as drivers for the model then the resulting motion will be specified by the whole body mass centre movement and whole body orientation time history. When a model is used in this way it is known as a simulation model.

Model Building

The human body is very complex with over 200 bones and 500 muscles and therefore any human body model will be a simplification of reality. The level of simplification of a simulation model will depend on the activity being simulated and the purpose of the study. For example a one-segment model of the human body may adequately represent the aerial phase of a straight dive but a model with two or three segments would be required for a piked dive to give an adequate representation. As a consequence a single model cannot be used to simulate all activities and so specific simulation models are built for particular tasks. As a general rule the model should be as simple as possible, while being sufficiently complex to address the questions set. This simple rule of thumb can be quite difficult to implement since the level of complexity needed is not always obvious.

Essentially forward dynamics simulation models can either be: [1] angledriven where the joint angle time histories are input to the model and the resulting whole body orientation and mass centre position are calculated (along with the required joint torques), or [2] torque / force driven where the joint torque / muscle force time histories are input to the model and the resulting kinematics are calculated. Angle-driven simulation models have typically been used to simulate activities which are not limited by strength such as the aerial phase of sports movements including diving (Miller, 1971), high jumping (Dapena, 1981), trampolining (Yeadon et al., 1990). They have also been used in other activities such as high bar circling (Yeadon and Hiley, 2000) or long swings on rings (Brewin et al., 2000) by limiting the joint torques in order to avoid unrealistic movements. Most force / torque driven simulation models have been used to represent relatively simple planar jumping movements where the human body can be represented using simplified planar 2D models. In addition movements where the body remains symmetrical about the sagittal plane such as swinging on rings (Sprigings et al., 1998) have often been modelled as this allows the simulation model to have fewer segments and hence fewer degrees of freedom.

Angle-driven models have typically been more complex with more segments and degrees of freedom as they are easier to control while torque driven models have been relatively simple in general due to the difficulties in determining realistic parameters for muscles. One notable exception is the jumping model of Hatze (1981a) which simulated the takeoff phase in long jumping. This model comprised 17 segments and 46 muscle groups but did not simulate the impact phase and did not allow for soft tissue movement.

Model Components

The following section will discuss the various components that are used to build a typical simulation model.

Linked segment models

Most of the whole body simulation models in sports biomechanics are based on a collection of rigid bodies (segments) linked together, and are generically called 'linked segment systems'. The rigid bodies are the principal building blocks of simulation models and can be thought of as representing the basic structure and inertia of the human body. For each rigid segment in a planar model four parameters are usually required: length, mass, mass centre location, and moment of inertia. The number of segments used depends on the aim of the study and the activity being modelled. For example: Alexander (1990) used a two-segment model to determine optimum approach speeds in jumps for height and distance, Neptune and Kautz (2000) used a planar twolegged bicycle-rider model to look at muscle contributions in forward and backward pedalling, and King and Yeadon (2003) used a planar five-segment model to investigate takeoffs in tumbling. The level of complexity needed is not always obvious. Torque-free two-segment models of vaulting have been used to show that the backward rotation generated during the takeoff of a Hecht vault is largely a function of the velocity and configuration at initial contact together with the passive mechanics during impact (King et al., 1999; Sprigings and Yeadon, 1997). These results were confirmed using a torquedriven five-segment model but it was also shown that the inclusion of a hand segment and shoulder elasticity made substantial contributions to rotation (King and Yeadon, 2005).

Wobbling masses

Although linked rigid body models have been used extensively to model many activities, a recent development has been to modify some of the rigid segments in the model by incorporating wobbling mass elements (Gruber et al., 1998). This type of representation allows some of the mass (soft tissue) in a segment to move relative to the bone (rigid part). For impact situations the inclusion of wobbling masses within the model is crucial as the loading on the system can be up to nearly 50% lower for a wobbling mass model compared

to the equivalent rigid segment model (Pain and Challis, 2006). The most common way to model wobbling masses is to attach a second rigid element to the first fixed rigid element (representing the bone) within a segment using non-linear damped passive springs with spring force $F = kx^3 - d\dot{x}$ where x is displacement and \dot{x} is velocity (Pain and Challis, 2001a).

The disadvantage of including wobbling mass elements within a simulation model is that there are more parameter values to determine and the equations of motion are more complex leading to longer simulation times. Wobbling mass segments should therefore only be included when necessary. Whether to include wobbling masses depends on the activity being modelled, although it is not always obvious whether they are needed. For example a simulation model of springboard diving (Yeadon et al., 2006) included wobbling mass segments, but when the springs were made 500 times stiffer the resulting simulations were almost identical.

Connection between rigid links

Typically the rigid links in the simulation model are joined together by frictionless joints whereby adjacent segments share a common line or a common point. For example Neptune and Kautz (2000) used a hinge joint to allow for flexion / extension at the knee while Hatze (1981a) used universal joint at the hip with three degrees of freedom to allow for flexion / extension, abduction / adduction and internal / external rotation. The assumption that adjacent segments share a common point or line is a simplification of reality and although reasonable for most joints, it is questionable at the shoulder where motion occurs at four different joints. Models of the shoulder joint have ranged in complexity from a one degree of freedom pin joint (Yeadon and King, 2002) to relatively simple visco-elastic representations (Hiley and Yeadon, 2003a) and complex finite element models (van der Helm, 1994). The level of complexity to be used depends on the requirements of the study. Simple visco-elastic representations have been used successfully in whole body models where the overall movement is of interest whereas complex models have been used to address issues such as the contribution of individual muscles to movement at the shoulder joint.

Interface with external surface

The simplest way to model contact between a human body model and an external surface, such as the ground or sports equipment, is to use a 'joint' so that the model rotates about a fixed point on the external surface (Bobbert et al., 2002). The disadvantage of this method is that it does not allow the model to translate relative to the point of contact or allow for a collision with the external surface since for an impact to occur the velocity of the point contacting the surface has to be non-zero initially. Alternatively forces can be applied at a finite number of locations using visco-elastic elements at the interface with the forces determined by the displacements and velocities of the points in contact. The visco-elastic elements can be used to represent specific elastic structures within the body such as the heel pad (Pain and Challis, 2001b) or sports equipment such as the high bar (Hiley and Yeadon, 2003b) or tumble track / foot interface (King and Yeadon, 2004). The equations used for the visco-elastic elements have varied in complexity from simple damped linear representations (King and Yeadon, 2004) through to highly non-linear equations (Wright et al., 1998). The number of points of contact varies but it is typically less than three (Yeadon and King, 2002) although 66 points of contact were used to simulate heel-toe running (Wright et al., 1998). The horizontal forces acting while in contact with an external surface can be calculated using a friction model (Gerritsen et al., 1995) where the horizontal force is expressed as a function of the vertical force and the horizontal velocity of the point in contact or by using visco-elastic springs (Yeadon and King, 2002). If visco-elastic springs are used the horizontal force should be expressed as a function of the vertical force so that the horizontal force falls to zero at the same time as the vertical force (Wilson et al., 2006).

Muscle models

Muscle models in sports biomechanics are typically based upon the work of A.V. Hill where the force-producing capabilities of muscle are divided into contractile and elastic elements (lumped parameter models) with the most commonly used version being the three-component Hill model (Caldwell, 2004). The model consists of a contractile element and two elastic elements

(series elastic element and the parallel elastic element). Mathematical relationships are required for each element in the muscle model so that the force exerted by a muscle on the simulation model can be defined throughout a simulation.

Contractile element

The force that a contractile element produces can be expressed as a function of three factors: muscle length, muscle velocity and muscle activation. The force-length relationship for a muscle is well documented as being bell-shaped with small tensions at extremes of length and maximal tension in between (Edman, 1992). As a consequence the force-length relationship is often modelled as a simple quadratic function.

The force-velocity relationship for a muscle can be split into two parts, the concentric phase and the eccentric phase. In the concentric phase tetanic muscle force decreases hyperbolically with increasing speed of shortening to approach zero at maximum shortening velocity (Hill, 1938). In the eccentric phase maximum tetanic muscle force increases rapidly to around 1.4 - 1.5 times the isometric value with increasing speed of lengthening and then plateaus for higher velocities (Harry et al., 1990; Dudley et al., 1990). Maximum voluntary muscle force shows a similar force-velocity relationship in the concentric phase, but plateaus at 1.1 - 1.2 times the isometric value in the eccentric phase (Westing et al., 1988; Yeadon et al., 2006).

The voluntary activation level of a muscle ranges from 0 (no activation) to 1 (maximum voluntary activation) during a simulation and is defined as a function of time. This function is multiplied by the maximum voluntary force given by the force-length and force velocity relationships to give the muscle force exerted. Ideally the function used to define the activation time history of a muscle should have a small number of parameters. One way of doing this is to define a simple activation profile for each muscle (basic shape) using a small number of parameters (Yeadon and King, 2002). For example in jumping the activations of the extensors rise up from a low initial level to a maximum level and then drop off towards the end of the simulation, while the flexor activations drop from an initial level to a low level and then rise towards the end of the simulation (King et al., 2006). These parameters are varied

within realistic limits in order to define the activation time history used for each muscle during a specific simulation.

Series elastic element

The series elastic element represents the connective tissue in series with the contractile element (tendon and aponeurosis). The force produced by the series elastic element is typically expressed as an increasing function of its length with a slack length below which no force can be generated. It is usually assumed that series elastic element stretches by around 5% at maximum isometric force (Muramatsu et al., 2001).

Parallel Elastic element

The effect of the parallel elastic element is often ignored in models of sports movements as this element does not produce high forces for the normal working ranges of joints (Chapman, 1985).

Torque generators vs. individual muscle representations

All simulation models that include individual muscle models have the disadvantage that it is very difficult to determine individual parameters for each element of each muscle, as it is impossible to measure all the parameters required non-invasively. As a consequence researchers rely on data from the literature for their muscle models and so the models are not specific to an individual. An alternative approach is to use torque generators to represent the net effect of all the muscles crossing a joint (e.g. King and Yeadon, 2002) as the net torque produced by a group of muscles can be measured on an isovelocity dynamometer. More recently the extensor and flexor muscle groups around a joint have been represented by separate torque generators (King et al., 2006). In both cases each torque generator consists of rotational elastic and contractile elements. Using torque generators instead of individual muscles gives similar mathematical relationships with the contractile element maximum voluntary torque produced expressed as a function of the muscle angle and muscle angular velocity (Yeadon et al., 2006).

Model construction

The following sections will discuss the process of building a simulation model and running simulations using the components described in the previous section.

Free body diagram of the model

A free-body diagram of a simulation model gives all the necessary information required to build the computer simulation model. The free-body diagram should include the segments, the forces and torques and the nomenclature for lengths (Figure 1). In the system shown there are two degrees of freedom since the two angles θ_a and θ_b define the orientation and configuration of the model.



Figure 1. Free body diagram of a two-segment model of a gymnast swinging around a high bar.

Generating the equations of motion

The equations of motion for a mechanical system can be generated from first principles using Newton's Second Law for relatively simple models with only a few segments (e.g. Hiley and Yeadon, 2003b). For a planar link model, three equations of motion are available for each segment using Newton's Second Law (F = ma) in two perpendicular directions and taking moments (T = $l\alpha$) for each segment. In Figure 1 this allows the calculation of one angle and two reaction forces for each segment.

For more complex models a computer package is recommended, as it can take a long time to generate the equations of motion by hand and the likelihood of making errors is high. There are a number of commercially available software packages (e.g. DADS, ADAMS, AUTOLEV and SD Fast) that can generate equations of motion for a user defined system of rigid and elastic elements. Each package allows the user to input a relatively simple description of the model and the equations of motion are then automatically generated, solved and integrated. Note, with all packages that automatically generate equations of motion it is important to learn how to use the specific software by building simple models and performing checks to ensure that the results are correct. Some packages (e.g. AUTOLEV) generate computer source code (typically Fortran or C) for the mechanical system. The advantage of this is that the user can then customise the basic simulation model to incorporate muscle models or an optimisation routine, for example. Other more complex packages do not give full access to the source code and this can prevent the model from being customised for specific tasks.

Model input and output

There are two sets of input that are required for a simulation to run. Firstly there are the initial kinematics which comprises the mass centre velocity, and the orientation and angular velocity of each segment. The initial kinematics can be obtained from recordings of actual performances, although it can be difficult to obtain accurate velocity estimates (Hubbard and Alaways, 1989). Secondly there is information required during the simulation. A kinematically driven model requires joint angle time histories (Yeadon, 1990a) while a kinetically driven model requires activation histories for each actuator (muscle or torque generator) in the model (Alexander, 1990; Neptune and Hull, 1999).

The output from both types of simulation model comprises time histories of all the variables calculated in the simulation model. For a kinematically driven model this is the whole body orientation, linear and angular momentum and joint torques, while for a kinetically driven model it comprises the whole body orientation, linear and angular momentum and joint angle time histories.

Integration

Running a simulation to calculate how a model moves requires a method for integrating the equations of motion over time. The simplest method to increment a set of equations of motion (ordinary differential equations) through a time interval dt is to use derivative information from the beginning of the interval. This is known as the 'Euler method' (Press et al., 1988):

$$x_{n+1} = x_{x} + \dot{x}_{n} dt + \frac{1}{2} \ddot{x}_{n} dt^{2}$$

The Euler method assumes a fixed step length of dt, where dt is equal to 0.0001s, for example. The disadvantage of the Euler method is that a comparatively small step size is needed and the method is not very stable (Press et al., 1988). A better method is to use a fourth order Runge-Kutta in which four evaluations of the function are calculated per step size (Press et al., 1988). In addition most good integration routines include a variable step size with the aim to have some predetermined accuracy in a solution with minimum computational effort (Press et al., 1988).

A kinetically driven model requires the force / torque produced by each actuator to be input to the model at each time step. The force / torque produced is a function of the actuators level of activation, length and velocity. The movement of the contractile element / series elastic element must therefore be calculated. Caldwell (2004) gives an in-depth account of this procedure, but essentially at each time step the total length of the actuator is split between the contractile element and series elastic element in such a way that the force / torque in each element are equal.

Error checking

Whatever method is used to generate the equations of motion, it is always important that checks are carried out to ensure that simple programming errors haven't been made. Example are: [1] Energy is conserved if all damping is removed and all the muscles are switched off; [2] The mass centre of the model follows a parabola if the forces between the simulation model and the external surface are set to zero; [3] Impulse equals change in linear momentum; [4] Angular momentum about the mass centre is conserved during flight.

Optimisation

Simulation models can be used to find the optimum technique for a specific task by running many simulations with different inputs. To perform an optimisation is a three stage process. Firstly an objective function (or performance score) must be formulated which can be maximised (or minimised) by varying inputs to the model within realistic limits. For jumping simulations the objective function can simply be the jump height (or jump distance) but for movements where rotation is also important a more complex function incorporating both mass centre movement and rotation is required. The challenge for formulating such an objective function is to determine appropriate weightings for each variable in the function since the weightings affect the solution.

Secondly realistic limits need to be established for each of the variables (typically activation parameters to each muscle and initial conditions). Additionally the activation patterns of each muscle need to be defined using a small number of parameters to keep the optimisation run time reasonably low and increase the likelihood of finding a global optimum.

Thirdly an algorithm capable of finding the global optimum rather than a local optimum is needed. Of the many algorithms available the Simplex algorithm (Nelder and Mead, 1965), the Simulated Annealing algorithm (Goffe et al., 1994) and Genetic algorithms (van Soest and Cassius, 2003) have proved popular. The Simplex algorithm typically finds a solution quickly but can get stuck at a local optimum as it only accepts downhill solutions, whereas the Simulated Annealing and Genetic algorithms are better at finding the global optimum as they can escape from local optima.

Summary of Model Building

- Decide what factors are important
- Decide upon the number of segments and joints
- Decide whether to include wobbling masses
- Draw the free body diagram showing all the forces acting on the system
- Decide whether the model is to be angle-driven or torque driven
- Decide which muscles should be represented
- Decide how to model the interface with the ground or equipment

 Decide whether to use a software package or to build the model from first principles

Parameter Determination

Determining parameters for a simulation model is difficult but vital as the values chosen can have a large influence on the resulting simulations. Parameters are needed for the rigid / wobbling mass segments, muscle-tendon complexes and visco-elastic elements in the model. Fundamentally there are two different ways to approach this, either to estimate values from the literature, or take measurements on a subject to determine subject-specific parameters. There is a clear advantage to determining subject-specific parameters as it allows a model to be evaluated by comparing simulation output with performance data on the same subject.

Inertia parameters

Accurate segmental inertia values are needed for each segment in the simulation model. For a rigid segment the inertia parameters consist of the segmental mass, length, mass centre location and moment of inertia (one moment of inertia value is needed for a planar model, while three moment of inertia values are needed for a 3D model). For a wobbling mass segment there are twice as many inertia parameters needed since a wobbling mass segment comprises two rigid bodies connected via visco-elastic springs.

There are two methods of obtaining rigid segmental inertia parameters. The first is to use regression equations (Hinrichs, 1985; Yeadon and Morlock, 1989) based upon anthropometric measurements and inertia parameters determined from cadaver segments (Chandler, 1975; Dempster, 1955). The disadvantage of this method is that the accuracy is dependent on how well the morphology of the subject compares with the cadavers used in the study. A better method which only requires density values from cadaver studies is to take anthropometric measurements on the subject and use a geometric model (Hatze 1980; Jensen, 1978; Yeadon, 1990b) to determine the segmental inertia parameters. Although it is difficult to establish the accuracy of these geometric models for determining segmental inertia parameters error values of around 2% have been reported for total body mass (Yeadon, 1990b).

An alternative method which is worthy of mention is to use medical imaging techniques (Martin et al., 1989; Zatsiorsky et al., 1990) to determine segmental inertia parameters. With current technology and ethical issues this approach is not a real alternative at present but in the future it might provide a means for determining subject-specific segmental density values or provide a means for evaluating other methods for determining subject-specific segmental inertia parameters.

Including wobbling mass segments within the model increases the number of unknown parameters that are needed for each segment. The combined segmental inertia parameters can be calculated using a geometric model or regression equations. However, the calculation of the inertia parameters of the separate fixed and wobbling masses requires additional information on the ratio of bone to soft tissue which is typically obtained from cadaver dissection studies (Clarys and Marfell-Jones, 1986). This ratio data can then be scaled to the specific subject using total body mass and percentage body fat (Pain and Challis, 2006; Wilson et al., 2006). In the future it may be possible to improve this method by determining the inertia parameters for the rigid and wobbling masses of each segment directly from medical imaging.

Strength parameters

Determining accurate subject-specific strength parameters for muscletendon complexes is a major challenge in sports biomechanics, which has resulted in two different ways to represent the forces produced by muscles. The first is to include all the major muscles that cross a joint in the simulation model as individual muscle-tendon complexes with the parameters for the individual muscles obtained mainly from animal experiments (e.g. Gerritsen et al., 1995). Although the parameters are sometimes scaled to a subject or group of subjects based upon isometric measurements (Hatze, 1981b) this method does not give a complete set of subject-specific strength parameters. The alternative approach is to use torque generators at each joint in the model to represent the effect of all the muscles around a joint (flexors and extensors represented by separate torque generators). The advantage of this approach is that the net torque at a given joint can be measured on an isovelocity

dynamometer over a range of joint angular velocities and joint angles for the subject and so subject-specific parameters can be determined that define maximal voluntary torque as a function of muscle angle and velocity (King and Yeadon, 2002; Yeadon et al., 2006). With this approach it is still necessary to use data from the literature to determine the parameters for the series elastic element for each torque generator. In recent studies (King et al., 2006) it has been assumed that the series elastic element stretches by 5% of its resting length during isometric contractions (de Zee and Voigt, 2001; Muramatsu et al., 2001). Although it would be desirable to be able to determine series elastic element parameters directly from measurements on the subject it has previously been shown that simulation results were not sensitive to these parameter values (Yeadon and King, 2002).

Visco-elastic parameters

Visco-elastic parameters are required for springs that are included within a simulation model (connection of wobbling masses, shoulder joint, foot (or hand) / ground interface and equipment). Sometimes these springs represent specific elements where it is possible to determine visco-elastic properties from measurements (Pain and Challis, 2001b) while in other models the springs represent more than one visco-elastic element and so make it much harder to determine the parameters from experiments (e.g. Yeadon and King, Visco-elastic parameters should ideally be determined from 2002). independent tests and then fixed within the model for all simulations (Pain and Challis, 2001a; Gerritsen et al., 1995). If this is not possible the visco-elastic parameters can be determined through an optimisation procedure by choosing initial values and then allowing the parameters to vary within realistic bounds until a optimum match between simulation and performance is found. With this method a torque-driven or angle-driven simulation model can be used, although it is easier to implement in an angle-driven model as the joint angle changes are specified and so there are less parameters to be determined. Optimising the parameter values has the potential for the springs to compensate for errors in the model. This can be overcome by using a small set of spring parameters, determining the parameters from more than one trial and then fixing the parameter values for the model evaluation. For

example Yeadon and King (2002) determined visco-elastic parameters for the interface between foot and tumble track from one trial using a torque-driven model and then evaluated the model on a different trial, while Yeadon et al. (2006) determined visco-elastic parameters for the interface of a diver with a springboard from four trials using an angle-driven model. Using more than one trial for determining the spring parameters also has the advantage that the model output should not be overly sensitive to the parameter values used.

Model Evaluation

Model evaluation is an essential step in the process of developing a simulation model and should be carried out before a model is used in applications. Although this step was identified as an important part of the process over 25 years ago (Panjabi, 1979) the weakness of many simulation models is still that the level of accuracy is unknown (Yeadon and Challis, 1994). While a number of models have been evaluated to some extent such as those of Hatze (1981a), Yeadon et al., (1990), Neptune and Hull (1998), Brewin et al., (2000), Yeadon and King (2002), Fujii and Hubbard (2002), Hiley and Yeadon (2003a) and King et al. (2006), many have not been evaluated at all.

The complexity of the model and its intended use should be taken into account when evaluating a model. For a simple model (e.g. Alexander, 1990) which is used to make general predictions it may be sufficient to show that results are of the correct magnitude. In contrast if a model is being used to investigate the factors that determine optimum performance in jumping, the model should be evaluated quantitatively so that the level of accuracy of the model is known (e.g. King et al., 2006). Ideally the model evaluation should encompass the range of initial conditions / activities that the model is used for with little extrapolation of the model to situations where the level of accuracy is unknown (Panjabi, 1979). For example if a simulation model of springboard diving is evaluated successfully for forward dives, the model may not work for reverse dives and so it should be also evaluated using reverse dives.

The purpose of model evaluation is to determine the accuracy which can then be borne in mind when considering the results of simulations. Furthermore a successful evaluation gives confidence that the model

assumptions are not erroneous and that there are no gross modelling defects or simulation software errors. Ideally the evaluation process should include all aspects of the model that are going to be used to make predictions. If a model is going to be used to investigate the effect of initial conditions on maximum jump height then the model should be evaluated quantitatively to show that for a given set of initial conditions the model can perform the movement in a similar way and produce a similar jump height. If a model is to be used to examine how the knee flexor and extensor muscles are used in jumping, the model should be evaluated to show that for a given jump the model uses similar muscle forces to the actual performance.

To evaluate a simulation model is challenging and may require a number of iterations of model development before the model is evaluated satisfactorily. Initially data must be collected on an actual performance by the sports Ideally this should be an elite performer who is able to work participant. maximally throughout the testing and produce a performance that is close to optimal. Time histories of kinematic variables (from video or an automatic system), kinetic variables (from force plate or force transducers) and EMG histories (if possible) should be obtained. Subject-specific model parameter values are then determined from the measurements taken on the subject (anthropometric, strength, etc) with as little reliance on data from the literature as possible (Yeadon et al., 2006; Wilson et al., 2006). The initial kinematic conditions (positions and velocities) for the model are then determined from the performance data and input to the model along with any other time histories that are required for the model to run a single simulation. If the model is kinetically driven this will consist of the activation time history for each actuator (Yeadon and King, 2002), while if the model is kinematically driven the time history of each joint angle will be required (Hiley and Yeadon, 2003a). Once a single simulation has been run a difference score should be calculated by quantitatively comparing the simulation with the actual performance. The formulation of the score depends on the activity being simulated, but it should include all features of the performance that the model should match (e.g. joint angle changes, linear and angular momentum, floor movement etc). The difficulty in combining severable variables into one score is that appropriate weightings need to be chosen for each part of the objective

function. For example Yeadon and King (2002) assumed that a 1° difference in a joint angle at takeoff was equivalent to a 1% difference in mass centre velocity at takeoff. Furthermore, for variables that cannot be measured accurately (e.g. wobbling mass movement) it may be more appropriate to add a penalty to the difference score if too much movement occurs (King et al., 2006). Finally the input to the model is then varied until the best comparison is found (score minimised) using an optimisation routine. If the comparison between performance and simulation is close (Figure 2) then the model can be used to run simulations. If not then the model complexity or model parameters need to be modified and the model re-evaluated. If the comparison gives a percentage difference of less than 10% this is often sufficient for applications in sports biomechanics.





Figure 2. Comparison of performance and simulation graphics for the tumbling model of Yeadon and King (2002).

Issues in Model Design

The design of a particular model should be driven by the intended use and the questions to be answered. For example if the aim is to determine the forces that act within the human body during running then an inverse dynamics model may be more appropriate than a forwards dynamics model. If the aim is to demonstrate some general mechanical principles for a type of movement then a simple model may be adequate. The issue of model complexity is not simple, however. While it is evident that simple models such as Alexander's (1990) model of jumping can give insight into the mechanics of technique, there is often a tendency to rely on the quantitative results without recourse to model evaluation. The issue of model evaluation for a simple model is problematic since all that can be realistically expected is ballpark or order of magnitude accuracy. In order to achieve anything approaching 10% accuracy when compared with actual performance a model of some complexity is usually required, comprising several segments, realistic joint drivers and elastic elements. The development of such a model is a nontrivial endeavour. Sprigings and Miller (2004) argue the case for "the use of the simplest possible model capable of capturing the essence of the task being studied", citing Alexander (1990) and Hubbard (1993) in support. The problem here is deciding at what point a model is too simple. If a model is so simple that it is 30% inaccurate then it is difficult to justify conclusions indicated by the model results unless they are robust to a 30% inaccuracy. It is evident that some measure of model accuracy is needed in order to reach conclusions.

Simple models of throwing in which the implement is modelled as an aerodynamic rigid body (Hubbard and Alaways, 1987) need to be complemented by a representation of the ability of the thrower to impart velocity in a given direction (Hubbard et al., 2001) in order that realistic simulations may be carried out. The same considerations apply to other models that do not include the human participant.

While a rigid body may be adequate for a model of equipment it is likely to be too simple for a model of an activity such as high jumping (Hubbard and Trinkle, 1985a, b) although a rigid body model has been used to give insight into the two general modes of rotational aerial motion (Yeadon, 1993a).

Joint angle time-histories are sometimes used as drivers for a simulation model. In the case of aerial movement (van Gheluwe, 1981;Yeadon et al., 1990) it can be argued that this is a reasonable approach so long as the angular velocities are limited to achievable values. In activities where there are large contact forces with the external surroundings this approach is more problematic since steps need to be taken to ensure that the corresponding joint torques are achievable. Hiley and Yeadon (2003a, b) and Brewin et al. (2000) used angle-driven models to simulate swinging on the high bar and on the rings and eliminated simulations which required larger torques than were achieved by the participant on an isovelocity dynamometer. Another approach is to use joint torques as drivers where the maximum voluntary joint torque is a function of angular velocity (Alexander, 1990) and possibly of joint angle (King and Yeadon, 2003). This approach leads to more realistic simulations than the use of angle-driven models but there is a corresponding loss of the simple control of joint angles. Finally there are models which use representations of individual muscles or muscle groups crossing a joint (Hatze, 1981a; Neptune and Hull, 1998) and these have the potential to provide even more accurate representations but pose the problem of determining appropriate muscle parameter values.

Reviews of computer simulation modelling are provided by Alexander (2003), Hubbard (1993), King (1984), Miller (1975), Vaughan (1984, 1989) and Yeadon (1987).

The Inverse Dynamics Problem

The inverse dynamics problem is to determine the forces that must act in order to produce a given motion. Theoretically the only information needed comprises the time histories of the variables that define the motion of the system. From a practical perspective, however, estimates of angular accelerations from the given data typically have large errors and so additional information is often provided in the form of recorded ground reaction forces. As an example a four-segment model representing a handstand on a force plate will be used to determining the torques acting at each joint.

An inverse dynamics model of a handstand

The body is represented by four rigid segments H, A, B, C representing the hands, arms, trunk+head, and legs (Figures 3, 4). Newton-Euler equations are used to generate three equations per segment for the six joint reaction forces, three angular accelerations and three joint torques. This system of 12 equations in 12 unknowns is reduced to a system of six equations in the joint accelerations and joint torques by eliminating the six reaction forces. A knowledge of the segmental inertia parameters of a gymnast together with the time histories of the three joint angles during a handstand then permits the calculation of the joint torque time histories.

Nomenclature

Н	:	hand segment
A	:	arm segment
В	:	body (trunk and head) segment
С	:	leg segment
J_1	:	wrist joint
J_2	:	shoulder joint
J ₃	:	hip joint
(x _i , z _i)	:	joint centre coordinates (i = 1,3)
Fi	:	horizontal joint reaction forces $(i = 1,3)$
Ri	:	vertical joint reaction forces $(i = 1,3)$
Р	:	centre of pressure
(x_j, z_j)	:	segment mass centre coordinates (j = h, a, b or c)
(x _p , z _p)	:	point of force application $(z_p \text{ is assumed} = 0)$
Xj	:	horizontal linear accelerations of segment mass centres (j = h, a, b or c)
Zj	:	vertical linear accelerations of segment mass centres (j = h, a, b or c)
lj	:	moment of inertia about segment mass centres (j = h, a, b or c)
фj	:	segment angular accelerations $(j = h, a, b \text{ or } c)$



Figure 3. Free body diagram for a four-segment model of a handstand.

Each of the four segments (H, A, B, C) produce three equations: one for resultant vertical force, one for resultant horizontal force, one for moments about the mass centre.

Hand (H) : (assumed stationary)



H :
$$-T_1 + R(x_p - x_h) + R_1(x_h - x_1) + F(z_h - z_p) + F_1(z_1 - z_h) = 0$$
 (3)



Body (B) :

$$\uparrow \qquad : \qquad R_{2} - R_{3} - m_{b}g = m_{b}\ddot{z}_{b}$$

$$- (7)$$

$$F_{3} \rightarrow \qquad : \qquad F_{2} - F_{3} = m_{b}\ddot{x}_{b}$$

$$F_{2} - F_{3} = m_{b}\ddot{x}_{b}$$

$$F_{3}(x_{3} - x_{b}) = I_{b}\ddot{\phi}_{b}$$

$$F_{3}(x_{3} - x_{b}) = I_{b}\ddot{\phi}_{b}$$

Leg (C):

$$\uparrow$$
 \uparrow $R_{3} - m_{c}g = m_{c}\ddot{z}_{c}$
 $r_{c}g = m_{c}\ddot{z}_{c}$
 $r_{c}g = m_{c}\ddot{z}_{c}$
 $r_{3} - m_{c}g = m_{c}\ddot{z}_{c}$
 $F_{3} = m_{c}\ddot{x}_{c}$
 $F_{3} = m_{c}\ddot{x}_{c}$
 $-(11)$
 $C: T_{3} + F_{3}(z_{c} - z_{3}) - R_{3}(x_{c} - x_{3}) = I_{c}\ddot{\varphi}_{c}$
 (12)

Combining equations (1)+(4)+(7)+(10) resolves forces vertically for the whole system to give:

$$\mathbf{R} - \mathbf{mg} = \mathbf{m}_{a} \ddot{\mathbf{z}}_{a} + \mathbf{m}_{b} \ddot{\mathbf{z}}_{b} + \mathbf{m}_{c} \ddot{\mathbf{z}}_{c} \qquad - (13)$$

Combining equations (2)+(5)+(8)+(11) resolves forces horizontally for the whole system to give:

$$F = m_{a}\ddot{x}_{a} + m_{b}\ddot{x}_{b} + m_{c}\ddot{x}_{c} - (14)$$

Substituting for values R_1 and F_1 in (3) is equivalent to taking moments about J_1 for H and gives:

$$T_1 = F(z_1 - z_p) + R(x_p - x_1) - m_h g(x_h - x_1)$$
(15)

Combining equations (15) and (6), substituting for R_2 and F_2 is equivalent to taking moments about J_2 for H and A and gives:

$$F(z_{2} - z_{p}) - R(x_{2} - x_{p}) + m_{h}g(x_{2} - x_{h}) + m_{a}g(x_{2} - x_{a})$$

= $T_{2} + I_{a}\ddot{\phi}_{a} + m_{a}\ddot{x}_{a}(z_{2} - z_{a}) - m_{a}\ddot{z}_{a}(x_{2} - x_{a})$ - (16)

Combining equations (16) and (9), substituting for R_3 and F_3 and taking moments about J_3 for H, A and B gives:

$$F(z_{3} - z_{p}) - R(x_{3} - x_{p}) + m_{h}g(x_{3} - x_{h}) + m_{a}g(x_{3} - x_{a}) + m_{b}g(x_{3} - x_{b})$$

$$= T_{3} + I_{a}\ddot{\phi}_{a} + I_{b}\ddot{\phi}_{b} + m_{a}\ddot{x}_{a}(z_{3} - z_{a}) + m_{a}\ddot{x}_{b}(z_{3} - z_{b}) - m_{a}\ddot{z}_{a}(x_{3} - x_{a}) - m_{b}\ddot{z}_{b}(x_{3} - x_{b}) - (17)$$

Combining equations (17) and (12) is equivalent to taking moments about P for the whole system and gives:

$$-mg(x - x_{p}) = I_{a}\ddot{\phi}_{a} + I_{b}\dot{\phi}_{b} + I_{c}\dot{\phi}_{c} - m_{a}\ddot{x}_{a}(z_{a} - z_{p}) - m_{b}\ddot{x}_{b}(z_{b} - z_{p}) - m_{c}\ddot{x}_{c}(z_{c} - z_{p}) + m_{a}\ddot{z}_{a}(x_{a} - x_{p}) + m_{b}\ddot{z}_{b}(x_{b} - x_{p}) + m_{c}\ddot{z}_{c}(x_{c} - x_{p}) - (18)$$

Therefore, eliminating reactions at joints has left six equations of motion.

Using the representation below, the geometric equivalents of segment mass centre linear accelerations can be obtained by differentiating the position values twice.



Figure 4. A four-segment model of a handstand.

By substituting the geometric equivalents in place of the linear acceleration terms in equations (13)-(18) and re-arranging terms, we obtain six linear equations in the following form to solve for six unknowns (T₁, T₂, T₃, ϕ_a , ϕ_b , ϕ_c).

$$\begin{split} A_{11}T_1 + A_{12}T_2 + A_{13}T_3 + A_{14}\ddot{\varphi}_a + A_{15}\ddot{\varphi}_b + A_{16}\ddot{\varphi}_c &= B_1 \\ A_{21}T_1 + A_{22}T_2 + A_{23}T_3 + A_{24}\ddot{\varphi}_a + A_{25}\ddot{\varphi}_b + A_{26}\ddot{\varphi}_c &= B_2 \\ A_{31}T_1 + A_{32}T_2 + A_{33}T_3 + A_{34}\ddot{\varphi}_a + A_{35}\ddot{\varphi}_b + A_{36}\ddot{\varphi}_c &= B_3 \\ A_{41}T_1 + A_{42}T_2 + A_{43}T_3 + A_{44}\ddot{\varphi}_a + A_{45}\ddot{\varphi}_b + A_{46}\ddot{\varphi}_c &= B_4 \\ A_{51}T_1 + A_{52}T_2 + A_{53}T_3 + A_{54}\ddot{\varphi}_a + A_{55}\ddot{\varphi}_b + A_{56}\ddot{\varphi}_c &= B_5 \\ A_{61}T_1 + A_{62}T_2 + A_{63}T_3 + A_{64}\ddot{\varphi}_a + A_{65}\ddot{\varphi}_b + A_{66}\ddot{\varphi}_c &= B_6 \end{split}$$

All of the terms held in the coefficients A_{11} through B_6 can be derived from video or force data at each instant in time. A linear equation solver is used to determine estimates for the six unknowns at each time instant.

However a number of the equation coefficients involve $\cos\phi_a$, $\cos\phi_b$, $\cos\phi_c$ which result in singularities in the calculated torques and angular accelerations around $\phi_j = 90^\circ$ (j = a, b, c). To avoid this problem a further three equations are added using video estimates e_1 , e_2 , e_3 of the angular accelerations $\ddot{\phi}_a$, $\ddot{\phi}_b$, $\ddot{\phi}_c$. These may be written as:

$$A_{44}\ddot{\phi}_{a} = A_{44} e_{1}$$

$$A_{55}\ddot{\phi}_{b} = A_{55} e_{2}$$
$$A_{66}\ddot{\phi}_{c} = A_{66} e_{3}$$

which match the coefficients of $\ddot{\phi}_a$, $\ddot{\phi}_b$, $\ddot{\phi}_c$ in the last three of the six previous equations. This gives an over-determined system of nine equations for the six unknowns and a least-squares equation solver results in solutions without singularities. The addition of the further three linear equations constrains the angular acceleration estimates returned by the solver to sensible values and consequently the torque values returned are also more stable (Figure 5).



Figure 5. Joint torque obtained by inverse dynamics using (a) six equation system and (b) nine equation over-determined system. (Reproduced from Yeadon, M.R. and Trewartha, G. 2003. Control strategy for a hand balance. Motor Control 7, p. 418 by kind permission of Human Kinetics)

In analysing movements with an impact phase inverse dynamics is more problematic since it is not possible to include wobbling masses within an inverse dynamics model. In such situations a constrained forward dynamics model is a better way to proceed.

Solving the inverse dynamics problem using forwards dynamics simulation

An alternative to inverse dynamics is to use a constrained (angle-driven) forward dynamics simulation model for solving the inverse dynamics problem. This method allows wobbling masses to be included within the model which can have a substantial effect on the joint torques calculated especially during impact situations (Pain and Challis, 2006). The disadvantage of using a forward dynamics formulation is that it may be necessary to optimise a number of model parameters in order to find a solution that matches the

actual performance. In addition it is not possible to take advantage of an over-determined system and accurate acceleration values are needed which can be almost impossible to calculate during impacts.

With a constrained forward dynamics planar model there are three degrees of freedom for whole body motion (horizontal and vertical translation of the mass centre and whole body orientation) along with 3 degrees of freedom for each wobbling mass segment in the model. Time histories of the joint angles and external forces are input to the model and the motion of the model is calculated along with the joint torques required to satisfy the joint angle changes. King et al. (2003) used this method to calculate the net joint torque at the knee for the takeoff phase in a jump for height. The peak knee torques calculated using quasi-static, pseudo inverse dynamics (no wobbling mass movement), and constrained forward dynamics were 747 Nm, 682 Nm and 620 Nm. Including segment accelerations resulted in lower peak values and the inclusion of wobbling masses resulted in a smoother knee torque time history. However, all the calculated peak knee torgues were in excess of the eccentric maximum that could be exerted by the subject, which was estimated to be 277 Nm from isovelocity experiments with the subject (Figure 6). King et al. (2003) conclude that the discrepancy requires further investigation but it is likely to be due to modeling the knee as a simple frictionless pin joint or may be a consequence of errors in the digitised data.



Figure 6. Knee joint torque calculated using pseudo inverse dynamics and constrained forward dynamics.

Applications

In this section various examples of modelling in sports biomechanics will be given in order to illustrate model implementation and address the problems of optimisation, and the control of sports movements.

Understanding the mechanics of sports technique

It is possible to use a simulation model to gain insight into the mechanics of sports technique. Kinematic and kinetic data on an actual performance may suggest that a particular technique is responsible for the outcome but without some method of quantifying contributions little can be concluded. With a simulation model the efficacy of various techniques may be evaluated and so give insight into what really produces the resulting motion. van Gheluwe (1981) and Yeadon (1993b, c) used angle-driven simulation models of aerial movement to investigate the capabilities of various contact and aerial twisting techniques. They found that twist could be produced in the aerial phase of a plain somersault using asymmetrical movements of arms or hips. Dapena (1981) used an angle-driven model of the aerial phase of high jumping to show how a greater height could be cleared by modifying the configuration changes during flight.

Simulation models have provided insight into the mechanics of technique in: the flight phase of springboard diving (Miller, 1971), circling a high bar (Aramptsis et al., 1999, 2001; Hiley and Yeadon, 2001), skateboarding (Hubbard, 1980), the curved approach in high jumping (Tan, 1997), and landings in gymnastics (Requejo et al., 2004). It is important, however, that a model is evaluated before it is applied since the insights gained may be into the (incorrect) model rather than into actual performance.

Contributions

Simulation models may be used to determine the contributions of various aspects to the overall performance by simulating the effect of what happens when an aspect is removed or when just one aspect is present. In order that a variable can be used to quantify "contributions" it is necessary that such measures are additive. For example in twisting somersaults the use of the final twist angle achieved as a measure can lead to problems since the sum of twist angles produced by a number of techniques is likely to be greater than the twist resulting from the concurrent use of all the techniques. Additionally technique in the latter part of a twisting somersault may be primarily directed towards stopping the twist rather than producing the twist. Because of these effects Yeadon (1993d) used the maximum tilt angle as a measure of the twist potential in a movement. The tilt angles calculated in this way were additive and could be sensibly referred to as contributions from various twisting techniques.

Brewin et al. (2000) used a model of a gymnast and the rings apparatus to determine the contributions of technique and the elasticity of the gymnast and rings apparatus to the reduction of loading at the shoulders. It was found that technique reduced the loading by 2.7 bodyweights while elasticity reduced the loading by 1.1 bodyweights resulting in the actual loading of 8.5 bodyweights. King and Yeadon (2005) used a five-segment model of a gymnast during vaulting takeoff to investigate factors affecting performance of the Hecht vault. It was found that shoulder torque made only a small contribution of 7° to the resulting rotation whereas shoulder elasticity contributed 50° to the rotation in flight.

Optimisation of sports technique

Since a single simulation of a sports movement might take around one second it is possible to run thousands of simulations in a single day. This opens the way to investigating optimised performance by means of a theoretical study. The technique used in a sports movement is characterised using a number of parameters and then an optimisation procedure is used to find the best set of parameter values that maximises or minimises some performance score.

At the simplest level this could involve determining the optimal initial conditions in a projectile event such as basketball (Schwark et al., 2004) or javelin (Best et al., 1995; Hubbard and Alaways, 1987). Similarly optimum bat swing trajectories can be determined for maximum baseball range (Sawicki et al., 2003). In such optimisations a relatively small number of parameters corresponding to one instant in time are optimised. In such

situations it is important to take account of the inter-dependence of release parameters arising from the characteristics of the human participant (Hubbard et al., 2001).

More challenging are dynamic optimisations in which the time history of sports technique is optimised. Typically this requires a large number of parameters to characterise the technique used. In the case of angle-driven models it is a relatively simple matter to ensure that anatomical constraints at the joints are not violated (Hiley and Yeadon, 2003a). For models driven by joint torques or muscle representations such constraints cannot be imposed directly but can be accommodated using penalties as part of the optimisation function (Kong et al., 2005).

The performance score of the sports skill could simply be the distance thrown (Hubbard, 1984) or the distance jumped (Hatze, 1983; Hubbard et al., 1989), the height jumped (Nagano and Gerritsen, 2001; Cheng and Hubbard, 2004), the amount of rotation produced (Hiley and Yeadon, 2005; Sprigings and Miller, 2004), power output (van den Bogert, 1994), fatigue (Neptune and Hull, 1999) or more complex combinations of performance variables (Gervais, 1994; Koh et al., 2003). While such an approach may work it is also possible that the optimum solution is sensitive to small variations in technique, leading to inconsistent performance. This issue of robustness to perturbations will be discussed in the section on Control.

Control of Sports Movements

If a technique produces a perfect performance then inevitably there will be deviations from this performance resulting from small errors in timing. If the magnitude of such timing errors is known then the performance error may be calculated or conversely the timing errors may be estimated from the performance error. Yeadon and Brewin (2003) estimated that timing errors were of the order of 15 ms for a longswing to a still handstand on rings.

Some movements, such as a hand balance on floor (Yeadon and Trewartha, 2003) or a non-twisting straight somersault (Yeadon and Mikulcik, 1996), may be inherently unstable and may require continual proprioceptive feedback control in order to be performed at all. Other movements, such as twisting somersaults (Yeadon, 2001, 2002), may require continual feedback

correction to prevent drift away from the targeted performance. Variation in approach characteristics in tumbling may be compensated for by modifications in takeoff technique using feedforward control but only if such variation can be estimated in advance with sufficient accuracy (King and Yeadon, 2003).

Variation in technique can also be coped with by adopting a technique that is relatively insensitive (robust) to perturbations (van Soest et al., 1994; King and Yeadon, 2004). In cases where the limits of timing a movement are close to being reached, such considerations may be the main driver for selecting technique (Hiley and Yeadon, 2003b).

Conducting a Study

The main steps in conducting a study using a simulation model are as follows:-

- Identification of the research questions to be addressed
- Design of the model with these aims in mind
- Model construction
- Data collection for model input and parameter determination
- Parameter determination
- Model evaluation
- Experimental design of simulations to be run
- Results of simulations
- Conclusions: answering the research questions

Reporting on a Study

The format for reporting on a study will depend to some extent on the intended readership but should reflect the main steps listed in the previous section. Figures should be used when presenting a description of the model, performance data, simulation output, and model evaluation comparisons. The structure of a report or paper is usually along the following traditional lines:-

- Introduction: background, statement of aims
- Methods: model design, parameter determination, data collection, evaluation
- Results: simulation output, graphs, graphics, tables

• Discussion: addressing the aims, limitations, conclusions

Summary

The use of simulation models in sport can give insight into what is happening or in the case of a failing model what is not happening (Niklas, 1992). Models also provide a means for testing hypotheses generated from observations or measurements of performance. It should be remembered, however, that all models are simplifications and will not reflect all aspects of the real system. The strength of computer simulation modelling for sports science support is that it can provide general research results for the understanding of elite performance. While there is also the possibility of providing individual advice using personalised models, most sports biomechanics practitioners are a long way from realising this at present.

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