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UTILISING NUTATION IN TWISTING SOMERSAULTS

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INTRODUCTION

The orientation of a rigid body may be specified by successive rotations about the body axes $\mathbf{f}_1$ (lateral), $\mathbf{f}_2$ (frontal), $\mathbf{f}_3$ (longitudinal) through angles ϕ (somersault), θ (tilt), ψ (twist). If $\mathbf{f}_1$ is initially aligned with the angular momentum vector $\mathbf{h}$, the somersault angle ϕ is the angle traced by the plane $\mathbf{f}_3\mathbf{h}$ about the fixed spatial axis $\mathbf{h}$, the tilt angle θ is the angle between $\mathbf{f}_3$ and the invariable plane perpendicular to $\mathbf{h}$, and the twist angle ψ is the angle of rotation about the body axis $\mathbf{f}_3$ (Figure 1).

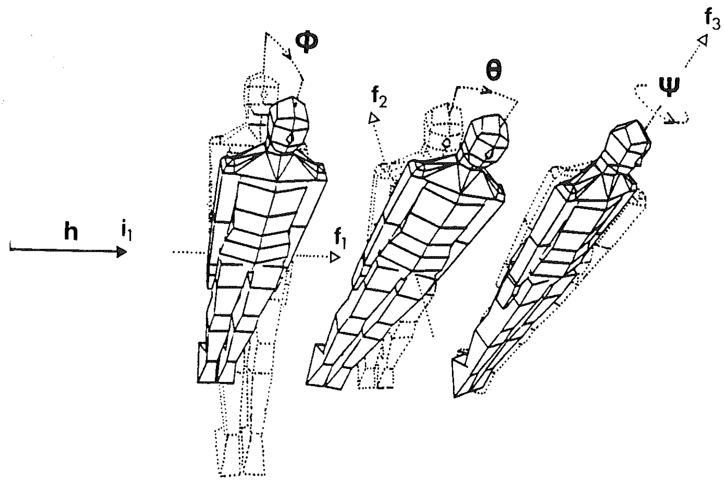

Figure 1: Angles of somersault (ϕ), tilt (θ) and twist (ψ).

The rotational motions of a rigid body in freefall comprise two distinct modes of motion: the wobbling mode in which the twist angle oscillates, and the twisting mode in which the twist angle increases monotonically (Yeadon, 1986). In the twisting mode the longitudinal axis nutates so that the tilt angle oscillates between values which are functions of the three principal moments of inertia. In a non-rigid body a change from one rigid configuration to another will result in a change of the amplitude of nutation. Since the rate of twisting is dependent upon the angle of tilt (Yeadon, 1993a), such a change may be used to increase the twist rate in twisting somersaults. This paper investigates how this may be achieved.

THEORY

The period of nutation in a twisting somersault is equal to the period of a half twist (Yeadon, 1993a). If A , B and C are the principal moments of inertia about axes through the mass centre of a human body in the straight position with $A > B > C$ then the corresponding principal axes will be $\mathbf{f}_2$, $\mathbf{f}_1$, $\mathbf{f}_3$. If $\psi = 0$ (the zero twist position) occurs when $\mathbf{f}_1$, $\mathbf{f}_3$ and $\mathbf{h}$ are coplanar, the corresponding tilt angle $\theta = \beta$ will be smaller than the tilt angle $\theta = \alpha$ occurring at $\psi = 90°$. Yeadon (1993a) shows that:

$$[1 - C/A]\cos^2\alpha = [1 - C/B]\cos^2\beta \qquad (1)$$

The increase in θ from β to α as the twist angle ψ increases from 0 to 90° will be greatest when the arms are abducted (wide arms). Let the values A_0, B_0, C_0, α_0 and β_0 correspond to this body configuration. The decrease in θ from α to β as ψ increases from 90° to 180° will be smallest when the arms are adducted (touching the body). Let the values A_1, B_1, C_1, α_1 and β_1 correspond to this configuration. Suppose that during a half twist the arms are rapidly adducted at the quarter twist position so that $\alpha_1 = \alpha_0$ and then rapidly abducted again at the half twist position. Two applications of equation (1) show that the tilt angle increases from β_0 to β_1 such that:

$$\cos\beta_1 = k.\cos\beta_0$$

where:

$$k^2 = \frac{[1 - C_1/A_1][1 - C_0/B_0]}{[1 - C_1/B_1][1 - C_0/A_0]}$$

Repeated application over successive half twists starting with $\beta_0 = 0$ and using appropriate values for the principal moments of inertia (Yeadon, 1993b) produces the values shown in Table 1.

Table 1: Increase in tilt angle using symmetrical arm movements.

twist [revolutions]	tilt [degrees]
0.0	0.0
0.5	9.9
1.0	13.9
1.5	17.0
2.0	19.6
2.5	21.8
3.0	23.8
3.5	25.7
4.0	27.4
4.5	29.0
5.0	30.5
10.0	42.0
25.0	61.6
50.0	76.9
200.0	89.9

The values shown in Table 1 are within one decimal place for any β_0 lying between 0 and 1°. If β_0 is close to zero the time for the first half twist to occur will be large. It can be seen that eventually the motion can be transformed from a non-twisting somersault into a non-somersaulting twist when the tilt angle reaches 90°.

SIMULATIONS

A computer simulation model of aerial movement (Yeadon et al., 1990) was used to obtain the somersault, tilt and twist time histories in a twisting somersault employing this technique of rapid arm adduction and abduction. The arm abduction angle (in degrees) was defined to be equal to $45[1 + \text{sign}(\sin 2\psi)]$ so that the arms were either completely abducted or completely adducted. Figure 2 displays the histories of tilt and twist as functions of somersault.

It can be seen that the tilt angle has a local minimum at the times for which the twist is an integral number of half revolutions. At such times the twist rate also has a local minimum. The maxima of the the tilt angle and twist rate occur at the quarter and three quarter twist positions. A graphics sequence of this simulation is presented in Figure 3.

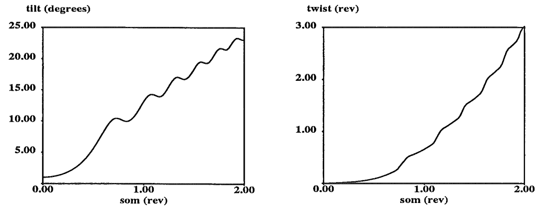

Figure 2: Tilt and twist as functions of somersault.

Figure 3: Simulation using instantaneous repeated arm adduction and abduction.

In this simulation it has been assumed that the arms may be moved instantaneously. If the maximum speed of arm movement is limited to 720° per somersault then after two somersaults the amount of twist produced is 2.1 rather than 3.0 revolutions and the tilt angle at the double twist position is 16.7° rather than 19.6°.

In practice the repeated use of symmetrical arm movement to produce tilt is not likely to be employed since it is less effective than asymmetrical arm movement. In a movement such as a full-out straight, in which there is little or no twist in the first somersault and a full twist in the second somersault, there is scope for using a single application of the nutation effect. Figure 4 shows a simulation of a full-out (straight body) using the proposed technique.

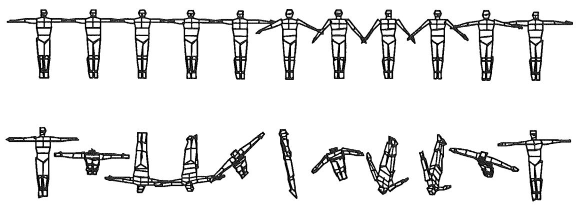

Figure 4: Simulation of a full-out straight.

The upper graphics sequence shows the arm movement employed. The lower sequence shows the effect that this produces in a double somersault in which the initial tilt angle is 1°. The arms are adducted from 90° to 40° around the quarter twist position and then abducted from 40° to 90° around the three quarter twist position. The tilt angle at the quarter twist position is 10.4° which becomes 9.9° at the three quarter twist position since the arm adduction around the quarter twist position is not instantaneous. The tilt angle at the half twist position is 6.6° which falls to 3.2° at the completion of the full twist due to the arm abduction near the three quarter twist position.

In Figure 5 a similar technique is used to produce two twists in the second somersault. The initial conditions are the same as for the previous simulation. Again the tilt angle falls near the completion of the two twists due to the arm abduction in the last phase. Since in both simulations the tilt angle has not been completely removed the body will still be twisting at the end of the movement. In practice this could be eliminated by using asymmetrical arm movements.

SUMMARY

A method has been identified for boosting the tilt angle in a twisting somersault by taking advantage of the nutation effect. By using symmetrical arm abduction and adduction it is theoretically possible to convert a non-twisting somersault into a non-somersaulting twist. The technique has practical application

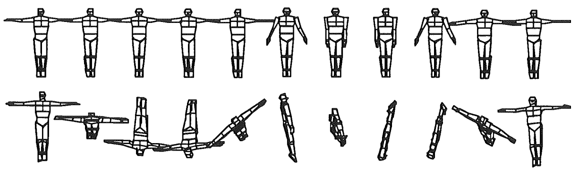

Figure 5: Simulation of a double-full-out straight.

to the full-out straight and double-full-out straight. These movements are used in dismounts from the gymnastics apparatus and in trampoline routines.

References

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