A NEW MATHEMATICAL MODEL OF FINGER COOLING USED TO PREDICT THE EFFECTS OF WINDCHILL AND SUBSEQUENT LIABILITY TO FREEZING COLD INJURY

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INTRODUCTION

Originally conceived for largely military applications', wind-chill indices are now commonly used and misused in a very large range of circumstances, and quoted in weather forecasts'. Original attempts to derive expressions of the cooling power of wind for given ambient temperatures were based on physical analogues, the katathermometer and frigorimeter[3]. It was Siple and Passell[1,4] who first produced curves for popular use and their study remains the basis for most wind-chill tables in current use, in spite of having a number of major flaws. Understanding of the physics involved in convective cooling has shown that models designed for one situation, such as the clothed whole body, are not applicable to others, such as naked fingers[3,5], although it is common to ignore this. An attempt in 1985 (Oakley, unpublished) to build a mathematical model of cooling in naked fingers, for use in the prediction of frostbite, appeared promising, and this paper reports continuing development of this model.

METHOD

For the purposes of this model, many simplifications and assumptions were made. In the first instance, the model is of a perfect cylinder which has one free end exposed to the environment. The other end does not gain or lose heat. The interior of the cylinder is divided into a number of concentric cylindrical layers in such a way that the inner cylinders do not touch the free end at the surface, but lie under their more superficial layers. The tissues within the layers are even in all respects, and parameters and variables uniformly and evenly distributed between layers, except with regard to blood flow. Axial transfer of heat is neglected in this second-generation model.

Heat production, gains and losses are computed for each discrete time period, and as a result the heat content of each layer is adjusted and its temperature recomputed for the start of the next time period. Each layer may produce heat by metabolism, dependent on temperature according to the Arrhenius relationship, may gain it by conduction from an adjacent layer, or may gain it from blood flowing through the layer. Thermal conductivity of tissue is varied between a minimum and a maximum according to an exponential relationship, as is blood flow. If the model is used in single layer mode, a chosen proportion of the heat available in the blood is given up uniformly to the whole tissue; if two or more layers are present, heat is delivered directly into one or more layers only. Heat loss from the outermost surface of the whole cylinder occurs by convection and radiation. Convective formulae offer free convection, dependent on the Grashof number, or forced convection, dependent on the Reynolds numbers, which in turn are related to the Nusselt number according to empirical relations for cylinders'. Radiative loss occurs according to Stefan's Law, to an assumed uniform ambient.

The model has been implemented in the Pascal language under MacApp® (Apple Computer, Inc.) on a Macintosh® IIcx computer. The finite differencing procedure is called as a result of interactively setting parameters and variables in a modeless dialog. Iteration is continued until a predetermined surface temperature (typically −0.55°C, the freezing point of tissue fluids) is reached. The results are delivered into a text document formatted for easy entry into popular spreadsheets, for further analysis. The model has been run using a range of different time intervals for the iteration, various numbers and thicknesses of layers, dimensional settings, and with output of layer temperatures and heat transfer by mode, as well as for different combinations of air temperatures and windspeed.

RESULTS

Convergent stability was found to occur with time increments of 0.5 s, which performed as well as increments down to 0.1 s. However, increments of 1.0 s and greater progressively diverged from those curves. Similar convergent stability with respect to the number of layers was found to occur with 3, 5 or greater than 5 layers, which produced remarkably similar cooling curves to those resulting from 12 or more layers (of the same total thicknesses). Dimensional effects were found to be very significant even for small (anthropometric) changes in length and diameter of the cylinder. At an ambient temperature of −30°C and a windspeed of 20 m/s, time to freezing varied by over 20% from a cylinder of length 90 mm and diameter 18 mm to one of length 70 mm and diameter 22 mm, and at −10°C and 5.0 m/s, this variation rose to 50%.

Examination of heat transfer by mode demonstrated that convective heat loss from the surface consistently accounted for the largest amount of heat transferred, with conduction to the outermost surface also being large.

Heat delivered by blood was significant only in the initial period of cooling, before blood flow fell to minimal levels, whilst conductive losses from the innermost layer rose inversely with blood flow. Radiative losses were relatively *small* and only slightly non-linear, whilst metabolic heat production was so small as to be insignificant at all times. Examination by layer exhibited a gradient through layers which steepened rapidly and non-linearly during cooling.

Predicted times to freezing (in s, for three layer model at 0.5 s increments):

Windspeed (m/s)	0.0	2.5	5.0	7.5	10	15	20	25	30
Temperature (°C)									
–5	∞	∞	∞	∞	∞	373	285	238	207
–10	∞	∞	432	303	242	179	144	122	106
–15	∞	497	277	204	165	121	97	81	70
–20	∞	357	210	155	124	91	72	60	51
–25	427	245	169	124	99	72	57	47	40
–30	334	238	142	104	83	59	47	39	33

∞ = rime in mess of 500 s; freezing not certain.

CONCLUSIONS

Whilst the performance of The model in terms of heat transfer by various modes and the temperame distribution between layers appears consistent with expectations, and the model appears to satisfy other internal validations, satisfactory external validation is more difficult Only one study[6] has been published which includes sufficient actual observational data to be useful for comparison, and there are substantial differences between the experimental conditions and those assumed for the model, and it no longer appears ethical to attempt to repeat such work. However, in each condition reported, the values predicted from the model lie within the observed range, and they are usually very close to the median, as shown below.

***predicted* and observed[6] times to freezing (in s):**

Windspeed (m/s)	5.0		10		15	
Temperature (°C)						
–15	277	352	*165*	90-168-∞	*121*	87-355-535
–25	*169*	49-85-310	99	43-152-285	72	–

Observed times given as medians between extremes, ∞ = did not freeze.

Comparison with the Siple and Passell[1] predictions is also difficult. However, the general form of the isotachs derived from this model is in accord with theirs, although the actual values are quite different. For instance, this model predicts that –10°C at 25 m/s is of equivalent risk as –25°C at 7.5 m/s, whilst Siple and Passell would offer a difference of about 15% (with the latter being the 'colder'). There is also controversy regarding the way in which wind-chill equivalent temperatures are expressed, in terms of the reference windspeed for no convective incremental loss[2], which the use of times in this model may circumvent.

This model offers insights into factors which determine the risk of *freezing* cold injury. For instance, the excess incidence of injuries in those of Negroid extraction when compared with Caucasians has been attributed to a combination of more rapid finger cooling and poorer subsequent vasodilation[8]. Substantial differences resulting from dimensional *changes* may be significant too, and future studies of this should include anthropometric factors. *The* next step in the development of this model is to incorporate axial heat transfer.

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