Design and Control of Advanced Functional Systems with Non-Conventional Material Properties

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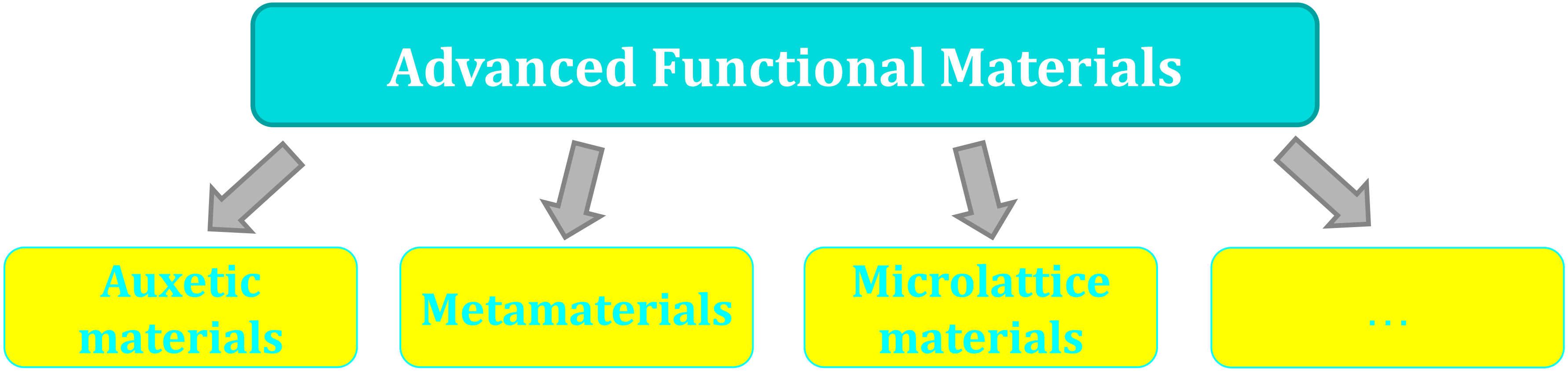
Structure

- Background
  - Advanced Functional Materials
  - Metamaterials (EPSRC growth area)
    - Electromagnetic/optic
    - Acoustic/Elastic/Mechanical
  - Advanced Functional Systems

- Systems challenges in metamaterials
  - Modelling and Systems Identification
  - Performance and Optimisation
  - Manufacture

Advanced Functional Material:

A collection of components/elements designed to provide a homogeneous material with a certain set of characteristics which would not arise through typical bulk/native material properties alone.



Materials with a negative poisson ratio

Materials with negative dynamic parameters, such as mass or permittivity

Materials with novel static properties, such as low mass to volume ratio.

Background to Metamaterials

- Metamaterials have received growing interest over the last 10-15 years due to their unique physical characteristics.
- Composed of arrays of sub-wavelength sized elements (meta-atoms) with effective dynamic bulk properties which can’t be found in nature.
  - Initially developed in Electromagnetics
  - Broadened to include Acoustics and Elastodynamics.
- They are currently listed by EPSRC as a research area to grow.

Background to Metamaterials

The dynamics of electromagnetic media are governed by Maxwell's equations (isotropic linear dielectric):

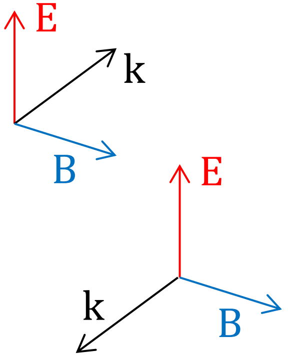
$$\nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t} \qquad B = \mu H$$

$$\nabla \times H = \frac{1}{c}\frac{\partial D}{\partial t} \qquad D = \varepsilon E$$

All known natural and conventional media are restricted to positive values for the constitutive parameters μ and ε.

Background to Metamaterials

Viktor Veselago in 1964 investigated the effect of negative μ and ε [1]:



$\mu > 0 \;\&\; \varepsilon > 0 \rightarrow E, H$ and k form a right–handed set

$\mu < 0 \;\&\; \varepsilon < 0 \rightarrow E, H$ and k form a left–handed set

The importance/interest of left–handed materials:

- Opposing phase and group velocity
- Negative refractive index $n = -\sqrt{\varepsilon\mu}$
- Reversed Doppler shift

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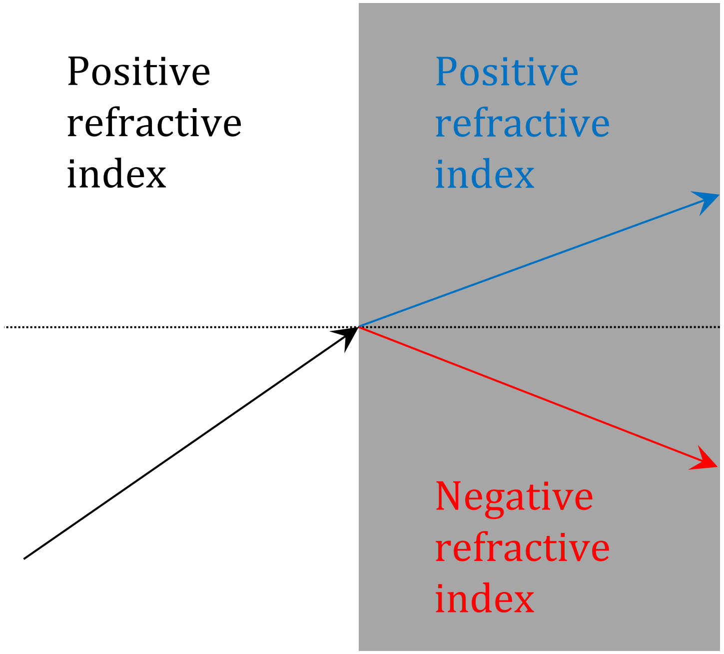
[1] V. Veselago, *Soviet Physics Uspekhi*, 10(4), 509-514, 1968.

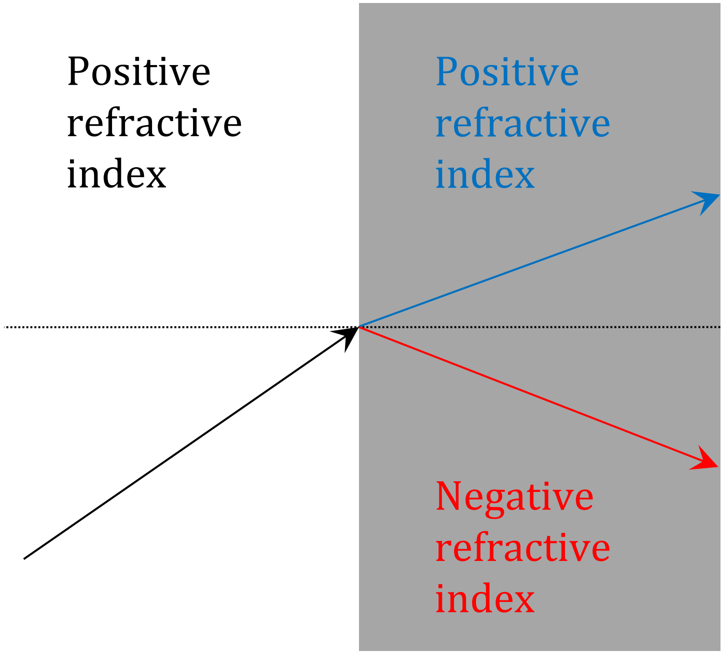
Background to Metamaterials

Negative refraction



Background to Metamaterials

Negative refraction → Applications

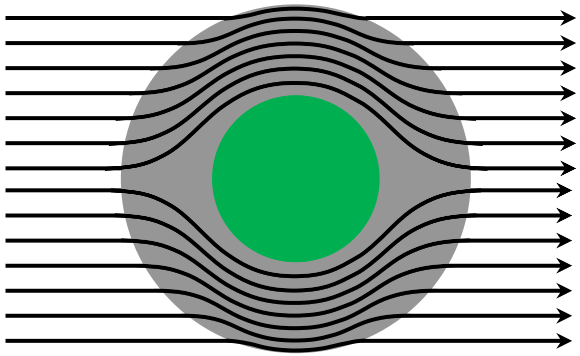


Positive
refractive
index

Positive
refractive
index

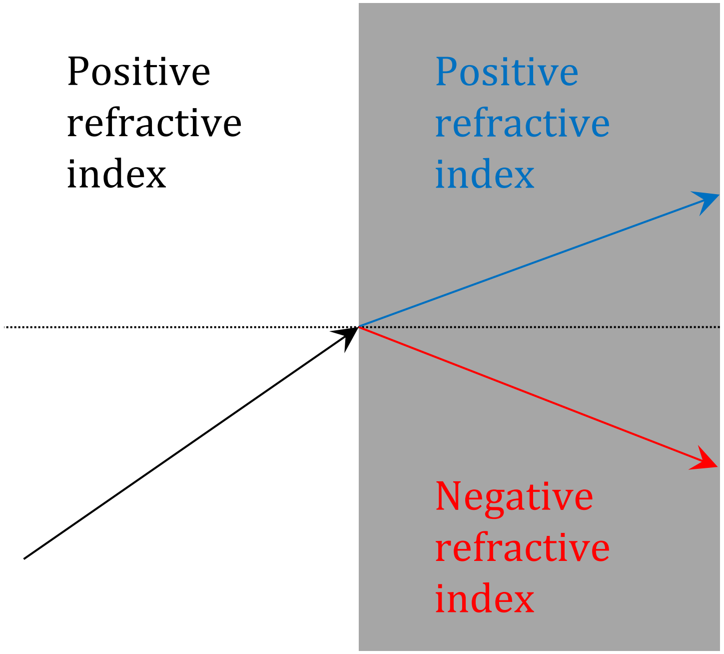
Negative
refractive
index

Invisibility cloaks



Background to Metamaterials

Negative refraction → Applications

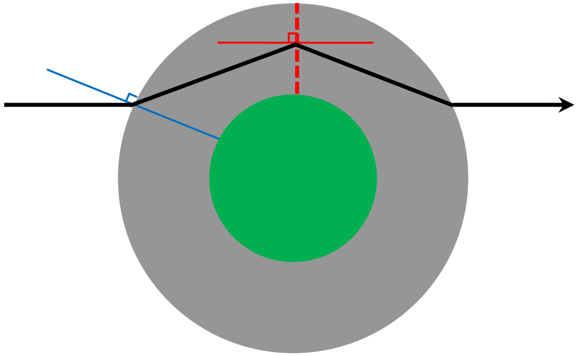


Positive refractive index

Positive refractive index

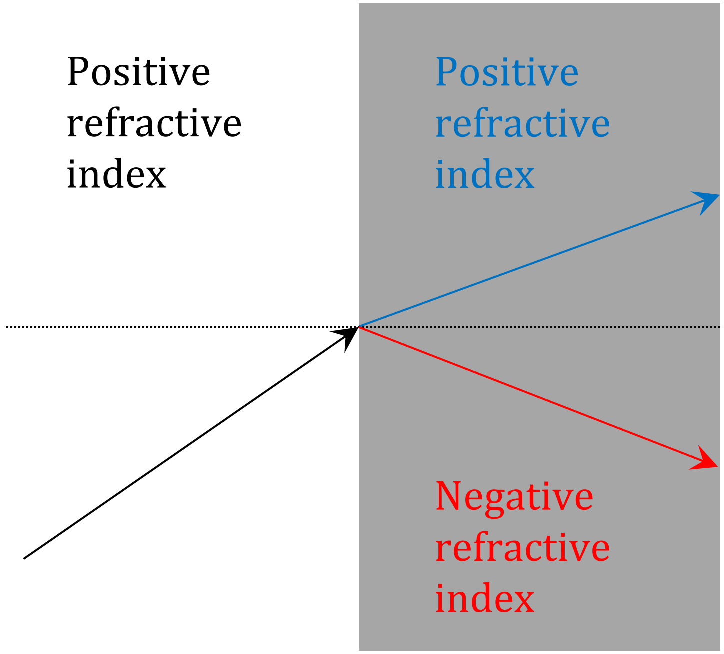
Negative refractive index

Invisibility cloaks



Background to Metamaterials

Negative refraction → Applications

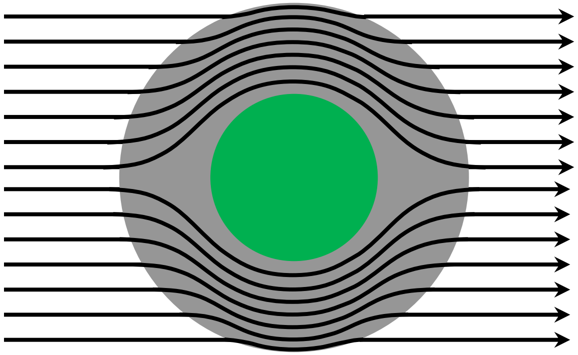


Positive refractive index

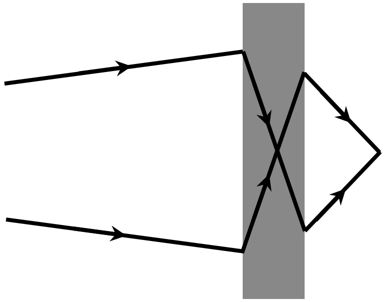
Positive refractive index

Negative refractive index

Invisibility cloaks



Flat lenses



Background to Metamaterials

What about $\mu > 0$ & $\varepsilon < 0$ or $\mu < 0$ & $\varepsilon > 0$?

- A material with permittivity and permeability with opposite signs has a complex refractive index.
- Such materials have a complex wave vector which efficiently blocks wave propagation in the material.

[2] D. R. Smith, J. B. Pendry, M. C. K. Wiltshire, *Science*, 305, 788, 2004.

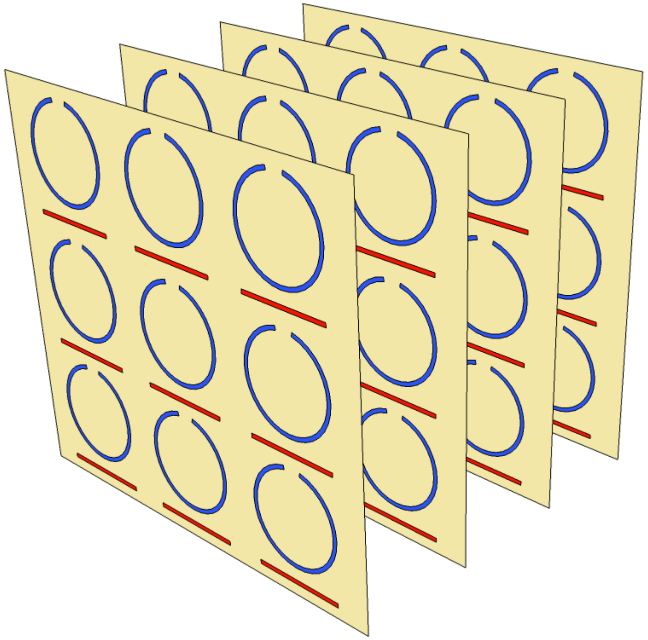
How can we achieve $\mu < 0$ & $\varepsilon < 0$?

- One answer is metamaterials
  - They are composed of a periodic array of sub-wavelength sub-structures.
- For example, an array of local resonators and the associated 180° phase change at resonance can lead to parameters (μ and ε) which are effectively negative for a homogeneous material.

When excited by an external electromagnetic field [2]:

Split ring resonators generate an oscillating magnetic field

Straight rod resonators generate an oscillating electric field



Acoustic and Elastic Metamaterials

The dynamics of elastic media are governed by Navier's equation (isotropic and homogeneous media):

$$\rho\frac{\partial^2\vec{u}}{\partial t^2} = \mu\nabla^2\vec{u} + (\mu+\lambda)\nabla(\nabla\cdot\vec{u}) + \vec{f}$$

Or the more general:

$$\rho\frac{\partial^2\vec{u}}{\partial t^2} = \nabla\cdot\vec{\sigma} + \vec{f}$$

$$\vec{\sigma} = C:\vec{\varepsilon}$$

$$\vec{\varepsilon} = \frac{1}{}[\nabla\vec{u} + (\nabla\vec{u})^T]$$

Acoustic and Elastic Metamaterials

- A problem similar to the electromagnetic case.
- However the general solution is more complex as any disturbance leads to both shear and pressure wave generation within an elastic medium.
- Much work considers the simpler case of acoustic wave propagation which is governed by:

$$\rho\frac{\partial^2\vec{p}}{\partial t^2} = \kappa\nabla^2\vec{p} + \vec{f}$$

Acoustic and Elastic Metamaterials

$$\rho \frac{\partial^2 \vec{p}}{\partial t^2} = \kappa \nabla^2 \vec{p} + \vec{f}$$

ρ – Density

κ – Bulk Modulus

$\rho > 0$ and $\kappa > 0$ in all natural/conventional media – which is intuitively obvious, otherwise for example gravity wouldn’t be much use in the conventional sense.

Acoustic and Elastic Metamaterials

The ideas for electromagnetic media can be extended to their mechanical counterparts, namely that:

If $\rho < 0 \;\&\; \kappa < 0$ the resulting media has:

- Opposing phase and group velocity
- Negative refractive index $n = -\sqrt{\frac{\rho}{\kappa}}$
- Reversed Doppler shift

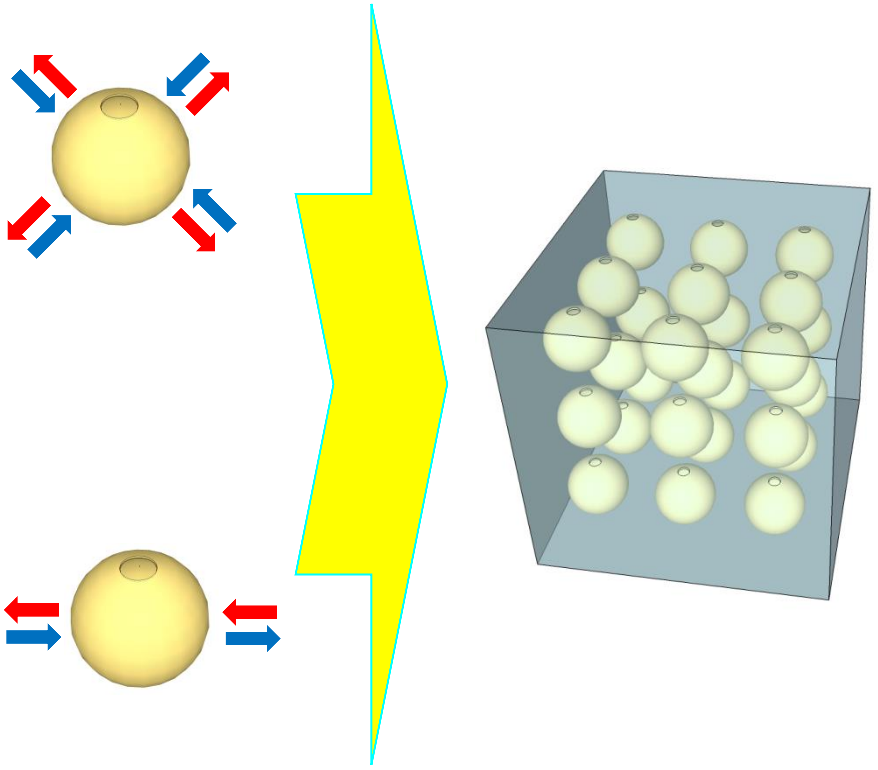
If $\rho < 0 \;\&\; \kappa > 0$ or $\rho > 0 \;\&\; \kappa < 0$ then propagating waves are effectively blocked and the material acts as a vibration/sound isolator.

Acoustic and Elastic Metamaterials

Applications

- Acoustic/Vibration cloaks:
  - Removing “dead spots” in rooms
  - Seismic protection for buildings
  - Strategic advantage – i.e. sonar evasion for submarines
- Imaging:
  - Ultrasound imaging for health and manufacturing
- Lightweight and thin high efficiency sound insulation
  - Buildings and transportation

How can we achieve $\rho < 0$ & $\kappa < 0$?



For example:

Negative Bulk Modulus – can result from a monopole resonance

The symmetrical expansion/contraction of a split hollow sphere provides such a resonance [3]

Negative Density – can result from a dipole resonance

The asymmetrical side-to-side (rigid body) resonance of a sphere [4] (or split hollow sphere) provides such a resonance

[3] C. Ding, L. Hao, X. Zhao, *J. Appl. Phys.* ,108, 074911, 2010.

[4] Z. Liu, X. Zhang, Y. Mao, Y.Y. Zhu, Z. Yang, C.T. Chan, P. Sheng, *Science*, 289, 1734, 2000.

How can we achieve $\rho < 0$ & $\kappa < 0$?

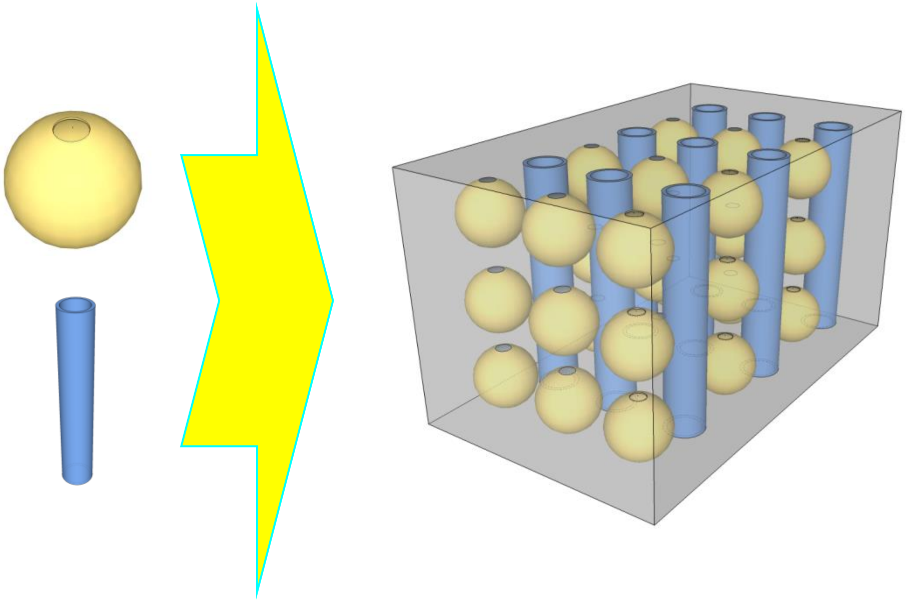
The required phase response for two different modes of the same element are unlikely to overlap.

More common to combine two different types of resonant elements.

For example [5]:

Split Hollow Sphere - **Negative Bulk Modulus**

Hollow rod - **Negative Density**



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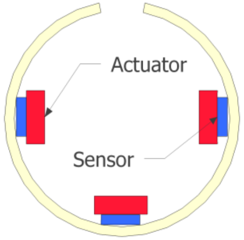
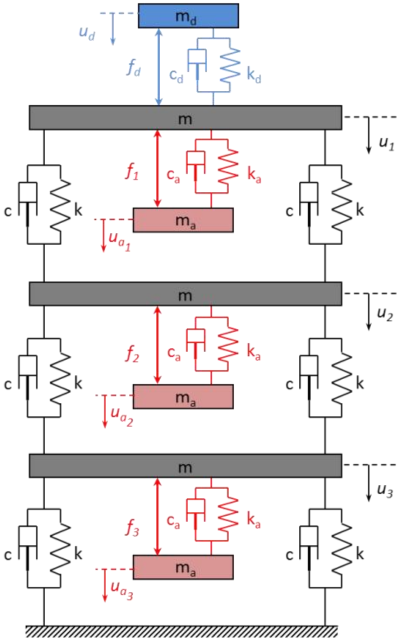
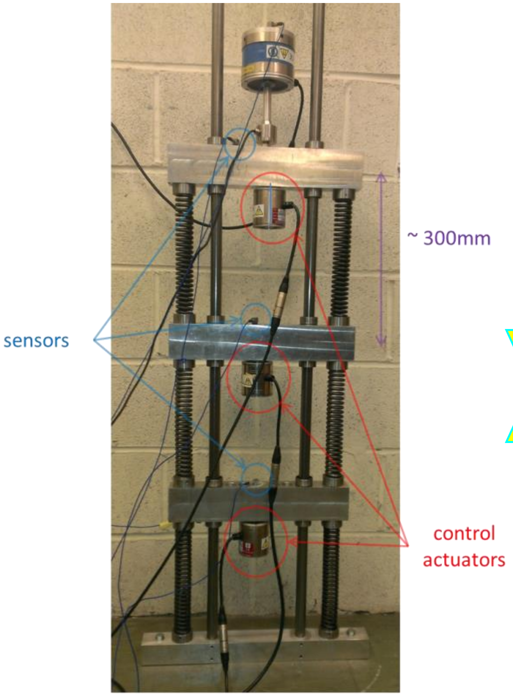
[5] H. Chen, H. Zeng, C. Ding, C. Luo, X. Zhao, *J. Appl. Phys.* ,113 (10), 104902, 2013.

[6] S. A. Pope, H. Laalej, “A Multi-Layer Active Elastic Metamaterial with Tuneable and Simultaneously Negative Mass and Stiffness”, *Smart Materials and Structures*, accepted.

Active Acoustic/Elastic Metamaterials[6]



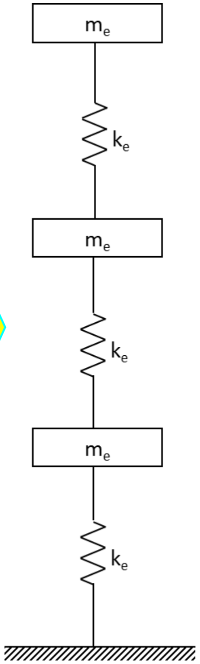


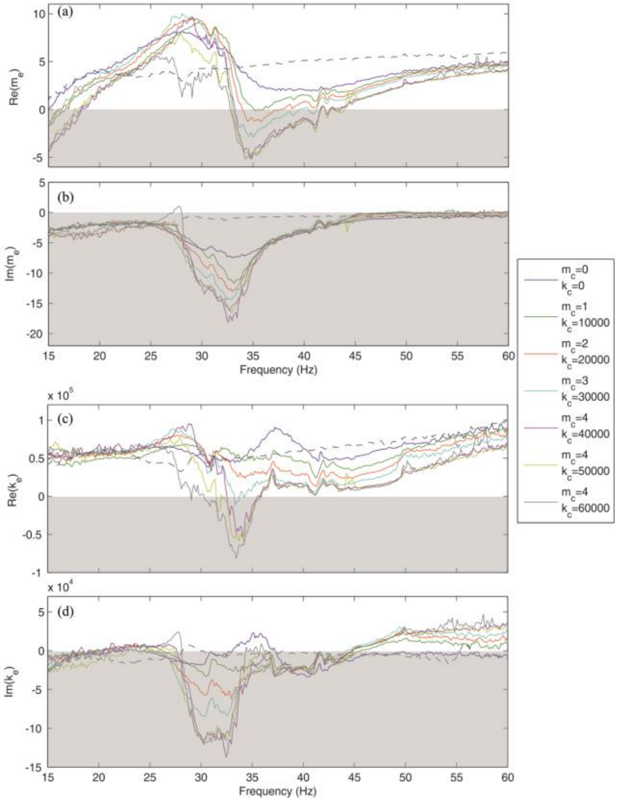


sensors

~ 300mm

control actuators





$$f_i = -m_c \ddot{u}_i + k_c(u_{i-1} + u_{i+1} - 2u_i)$$

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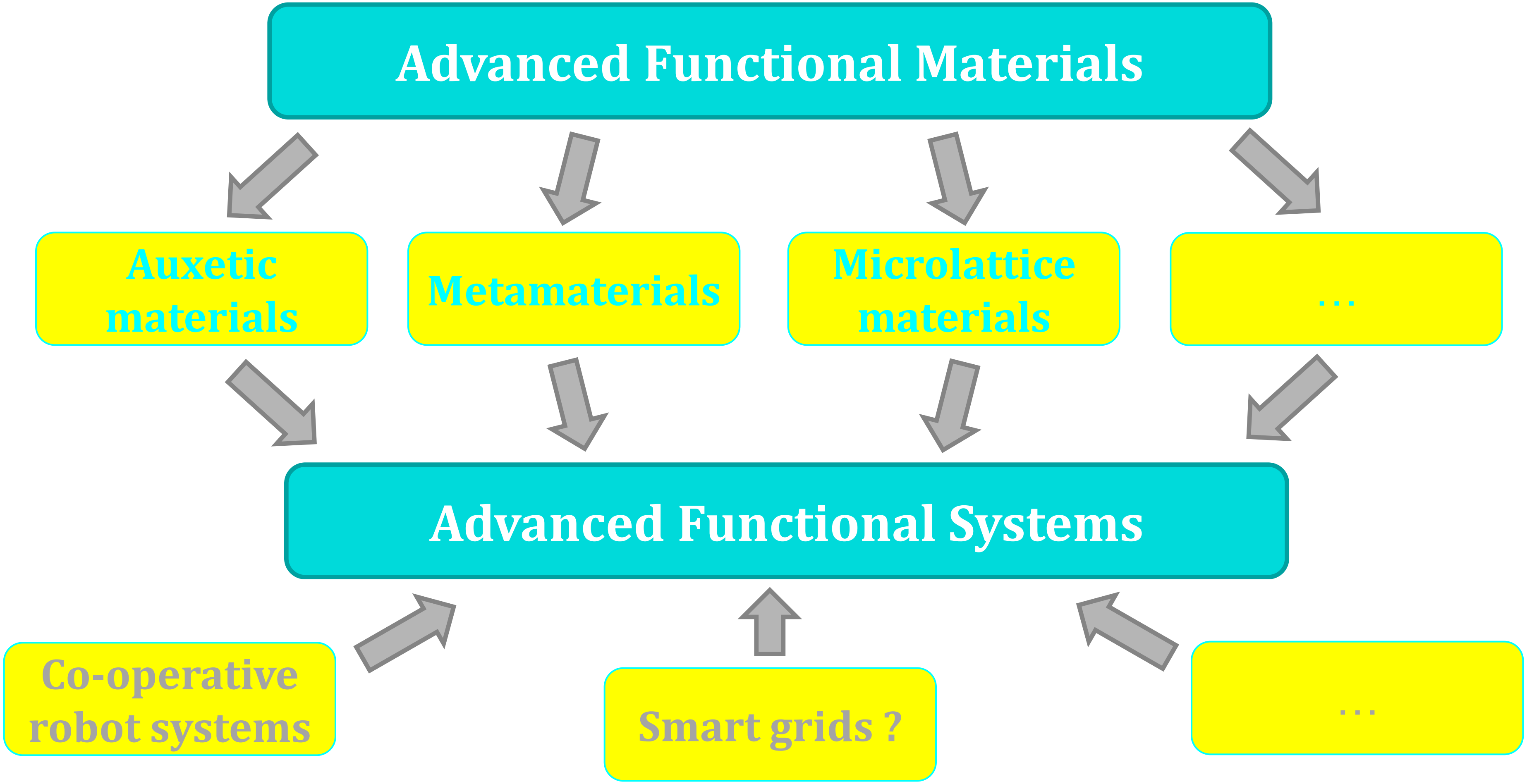
Advanced Functional System:

A collection of components or sub-systems designed to provide a homogeneous system with a certain set of characteristics which would not arise through the typical properties of a single (bulk) system (material) alone.



Advanced Functional Materials

Auxetic materials

Metamaterials

Microlattice materials

…

Advanced Functional Systems

Co-operative robot systems

Smart grids ?

…

Summary – Metamaterials

Domains

- Optical/Electromagnetic
- Acoustic
- Elastic/Mechanical
- Thermal

Types

- Resonant/Non-Resonant
- Passive/Active
- Linear/Non-Linear
- …

Summary – Metamaterial properties

- Single negative parameter
- Double negative parameter
- Polarisable (Chiral)
- Parameter anisotropy
- Unidirectional wave propagation (Rectification)
- Frequency selective and parameter tuneable
- Pentamode - solids that behave like fluids over a finite domain
- …

Systems Challenges for Metamaterial Research

Primarily driven by the need to translate fundamental work into application and industrial domains

- Shielding/Absorbers – including lightweight devices
- Cloaking – optical/seismic
- Lensing – flat and super-lenses
- Filtering
- Small antennas
- Improved sensing
- …

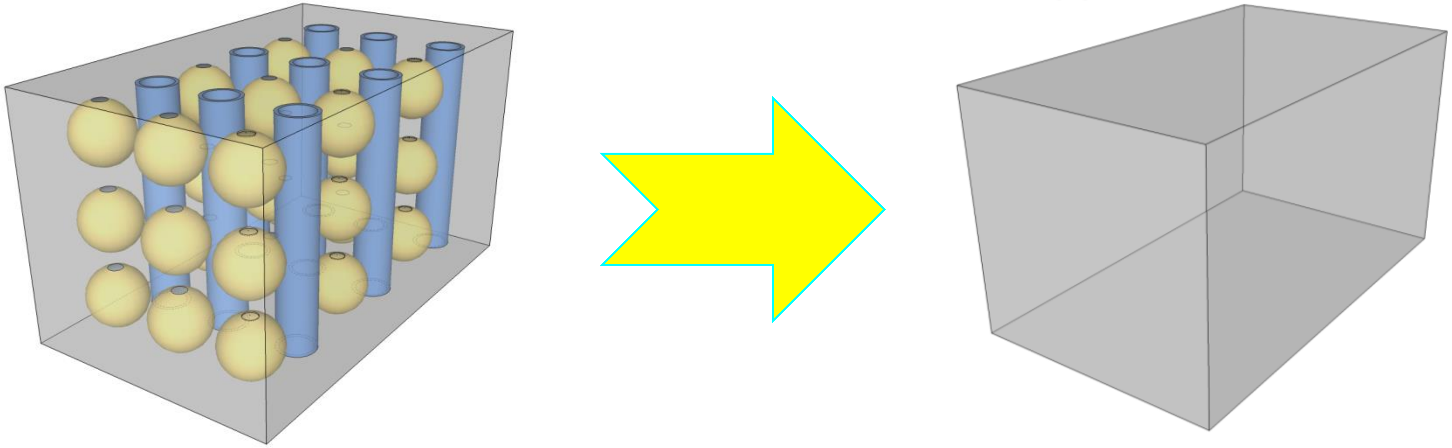
Systems Challenges for Metamaterial Research: **Modelling**

Homogenisation

- Large arrays of discrete elements with a complex local response → *What is their equivalent homogenous function?*
- Further complicated by the potential active, electromechanical and non-linear nature of the discrete systems/elements.

Characterised by a mixture of local/global and discrete/distributed variables and parameters

$$\rho_e \frac{\partial^2 \vec{p}(x,y,z)}{\partial t^2} = \kappa_e \nabla^2 \vec{p}(x,y,z)$$

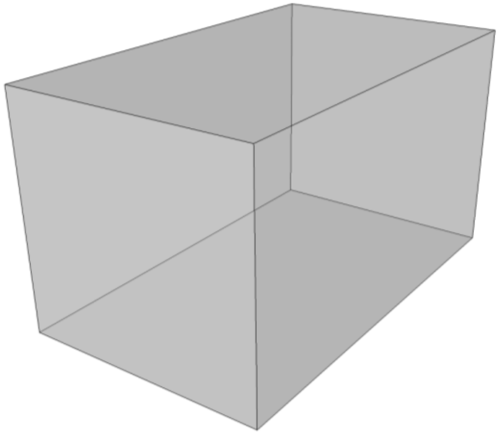


Systems Challenges for Metamaterial Research: **Modelling**

Spatiotemporal modelling

- Some applications require a complex spatial (including anisotropic) distribution for the effective material parameters
- These can also be time varying, particularly in adaptable materials.

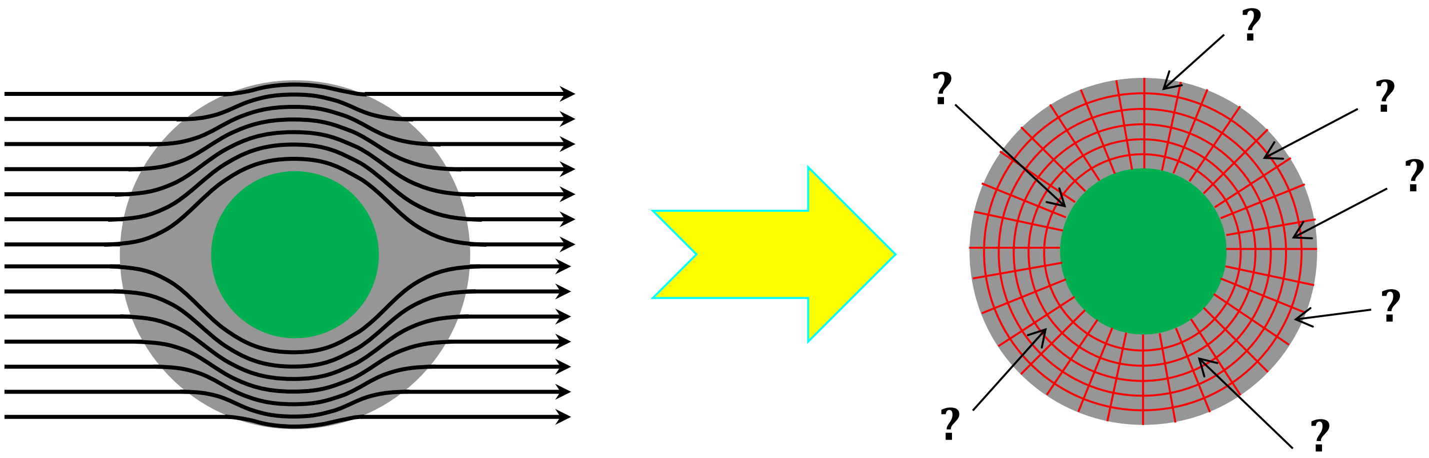
$$\rho_e(x,y,z,t)\frac{\partial^2\vec{p}(x,y,z)}{\partial t^2} = \kappa_e(x,y,z,t)\nabla^2\vec{p}(x,y,z)$$



Systems Challenges for Metamaterial Research: <u>**Modelling**</u>

Inverse modelling

- The desired outcome function might be known – for example the spatial distribution of the magnitude of the material parameters.
- *What is the required distribution of discrete elements and the their local properties?*

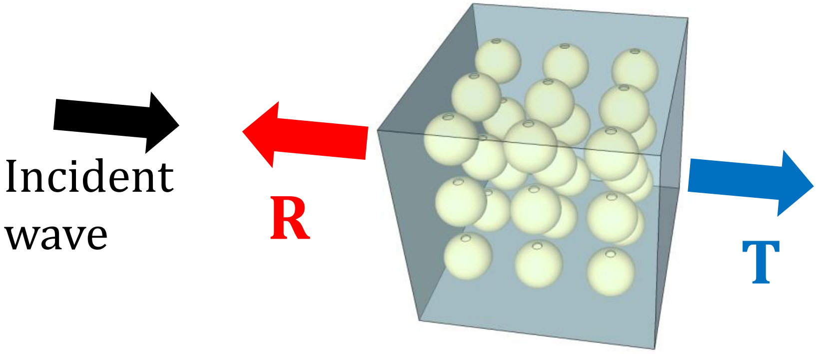


Systems Challenges for Metamaterial Research: **Systems Identification**

Objective is to manipulate the effective parameters of a system → *What are the characterising homogenous function and parameters of an experimental metamaterial?*

- A commonly used approach - measure the transmission and reflection coefficients, assume a standard linear function for the impedance and refractive index and solve for the effective parameters [7].

This is subject to certain conditions and for example cannot be applied to active materials or shear elastic waves.



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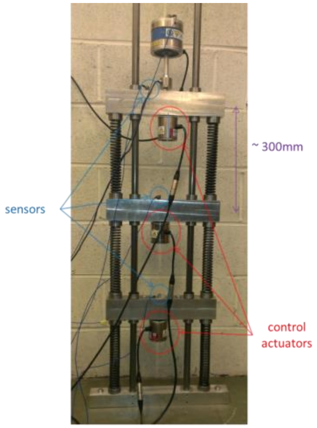
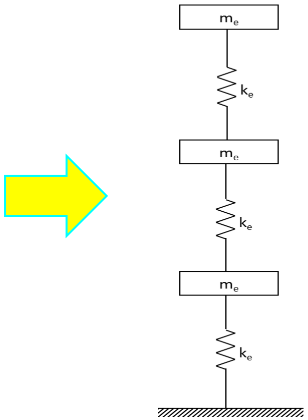
[7] V. Fokin, M. Ambati, C. Sun, X. Zhang, *Phys. Rev. B,* 76, 144302, 2007

Systems Challenges for Metamaterial Research: **Systems Identification**

- Another approach requires direct measurements of each discrete element, from which a homogenous model can be determined.
  - Advantage include that the global response is considered and it can be extended to a complete black box model, but a disadvantage is that it requires multiple internal measurements [6].





$$\begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix} T_1 & \omega^{-2}\left(T_2\left(j\omega\right)-T_1\left(j\omega\right)\right) \\ T_2 & \omega^{-2}\left(T_1\left(j\omega\right)+T_3\left(j\omega\right)-2T_2\left(j\omega\right)\right) \\ T_3 & \omega^{-2}\left(T_2\left(j\omega\right)-2T_3\left(j\omega\right)\right)\end{bmatrix}\begin{bmatrix}m_e\\k_e\end{bmatrix}$$

$$\boldsymbol{y} = \mathbf{X}\boldsymbol{\beta}$$

$$\boldsymbol{\beta} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\boldsymbol{y}$$

A need to identify linear and non-linear parameter functions for a wide range of designs and based on a wide range of measurements

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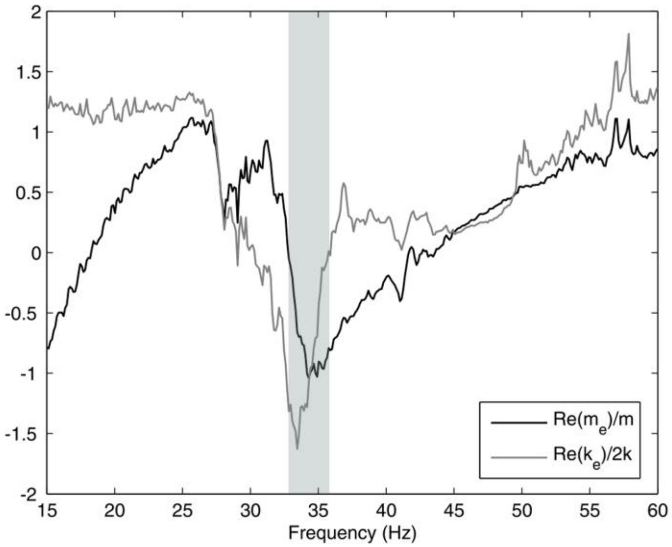
[6] S. A. Pope, H. Laalej, "A Multi-Layer Active Elastic Metamaterial with Tuneable and Simultaneously Negative Mass and Stiffness", *Smart Materials and Structures*, accepted.

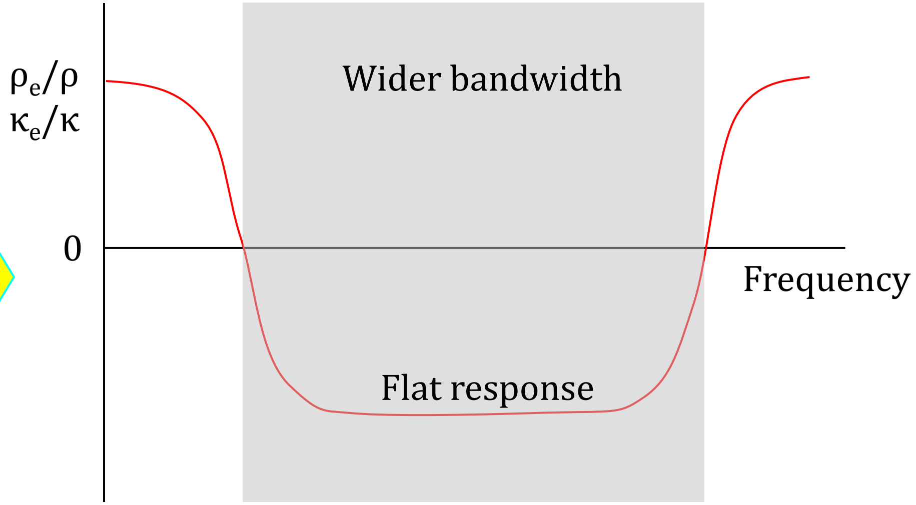
Systems Challenges for Metamaterial Research: **Improved Performance**

Bandwidth

The effective parameters are usually inherently dispersive → desired parameters are only present over a finite (and often narrow) bandwidths.

How can a larger bandwidth be achieved which is suitable for applications?



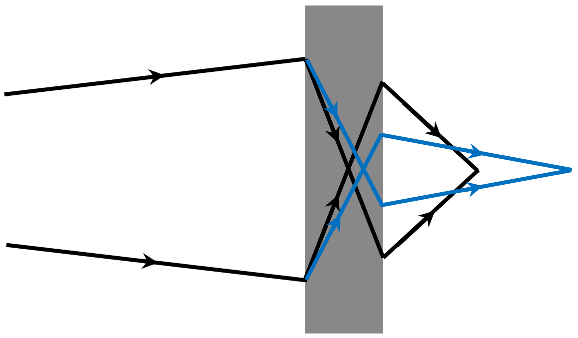


Systems Challenges for Metamaterial Research: **Improved Performance**

Tuning/Adaption

How can changing user or environmental demands be met?

e.g. variable focal length through control of the refractive index

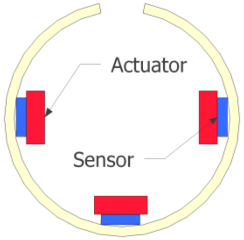


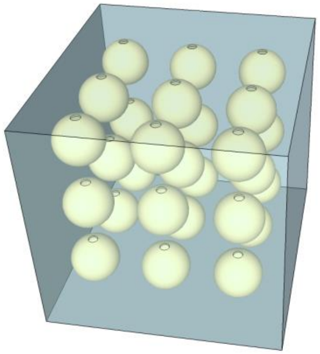
Passive/Active

There are clear pros and cons of either passive or active designs – *what is the best combination?*

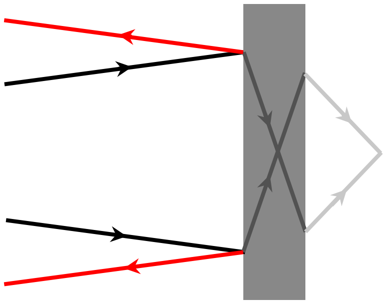
Inherently multi-input, multi-output systems

Can a standard design meet the requirements for a range of applications?





Systems Challenges for Metamaterial Research: **Improved Performance**



Impedance matching

Minimising reflection at the boundaries can be difficult in fluid/acoustic/elastic domains

Can the impedance match inherently required by some applications (e.g. cloaks and lenses) be effectively achieved?

Loss minimisation

Passive designs are inherently dissipative

Can the low transmission loss required by some applications (e.g. lenses) be achieved?

The problem of meeting the varied performance requirements is inherently a multi-objective optimisation problem:

spatial + temporal + frequency distribution of the parameters

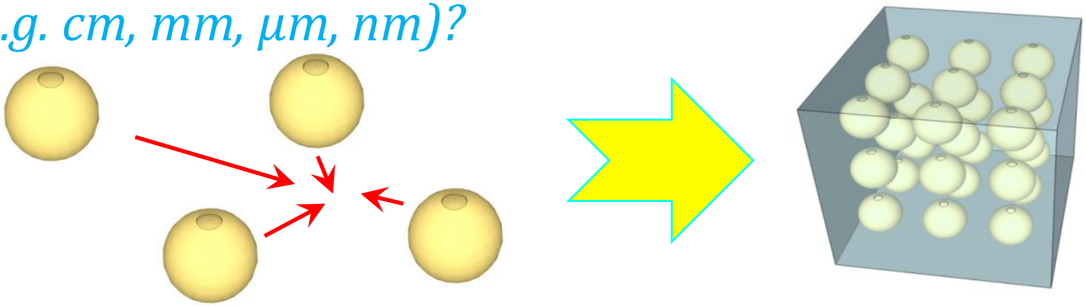
Systems Challenges for Metamaterial Research: **Manufacture**

Translating designs into devices

- The vast majority of work concentrates on producing designs to realise a particular parameter function or prototype application.
- *Can these designs be translated into market ready products?*
    - Design tools, manufacturing techniques, etc.

Self-Assembly

- Multiple small systems → *Is self assembly possible across a suitability wide range of scales (e.g. cm, mm, μm, nm)?*



Conclusion

- Metamaterials are part of an exciting new class of material
- To realised their potential it is important to consider them as Advanced Functional Systems.
- Systems Challenges in metamaterials research includes:
    - Modelling and simulating complex heterogeneous systems
    - Extracting the parameter functions for experimental implementations of these complex heterogeneous systems
    - Improving and tailoring the performance of what are inherently multi-objective and multi-input, multi-output systems
    - Manufacture and product development