Loughborough University

Department of Mathematical Sciences

MATHEMATICAL CHALLENGE

CHRISTMAS - 2014

Problem 1. Consider the sequence (x_n) defined recursively by

$$3^n x_n = 2^n x_n x_0 + 2^{n-1} x_{n-1} x_1 + \ldots + 2 x_1 x_{n-1} + x_0 x_n, \qquad n \geq 2,$$

with $x_0 = 1$, $x_1 = a$, where a is a real number.
Find the value of n for which x_n is maximal, depending on the value of a. Justify your answer.

Problem 2. For which m and n can one split the natural numbers from 1 to $N = mn$ into m groups with n numbers each such that all groups have the same sum? Justify your answer.
In particular, find such a splitting whenever it is possible for:
a) $N = 36$, $m = 9$ and $m = 12$;
b) $N = 35$, $m = 5$ and $m = 7$.

Problem 3. Ali Baba discovers a cave with 15 bags containing 1, 2,..., 15 golden coins respectively. He is keen to take a bag with as many coins as possible. Before that he is allowed to double the content of any bag from another bag containing sufficient coins to do that. He can repeat this operation as many times as he wants.
What is the maximal amount of coins Ali Baba can take home with him? Justify your answer.

Remarks.

1. There will be a first prize of £50 to the person handing in what will be considered to be the best effort to these problems. There may also be special prizes for the most original solutions.
2. Any student registered on one of the undergraduate programmes in the Department of Mathematical Sciences may submit solutions to any or all of these problems.
3. Solutions should be handed in on or before Friday 30 January 2015 to either Prof. A.P. Veselov (W233ma) or Dr. B. Winn (W278), who will be the judges for the Challenge.