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Loughborough University

Department of Mathematical Sciences

MATHEMATICAL CHALLENGE

CHRISTMAS - 2013

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Problem 1. Find a representation of $F(x) = x^4$ as the difference

$$F(x) = P(x) - Q(x)$$

of two monotonically increasing polynomials $P(x)$ and $Q(x)$.
Prove that such representation exists for any polynomial $F(x)$.

Problem 2. Define an infinite sequence of words by the following procedure. Starting with the word A at each step letter A is replaced by ABB, while B is replaced by BA :

$$A \to ABB \to ABBBABA \to ABBBABABAABBBAABB \to \ldots$$

Prove that the limiting infinite word is not periodic, but any finite part of it is repeated infinitely many times.
What is the proportion of letters A in this word?

Problem 3. In the company Amigo any two people who do not know each other have exactly two common friends, while any two people knowing each other have no common friends (it is assumed that people who know each other are friends). Alice and Bob claim that they have exactly 5 and 7 friends respectively. Prove that at least one of them is wrong.
Show that there could be such a company with 16 people, where everybody has exactly 5 friends.

Remarks.

1. There will be a first prize of £50 to the person handing in what will be considered to be the best effort to these problems. There may also be special prizes for the most original solutions.
2. Any student registered on one of the undergraduate programmes in the Department of Mathematical Sciences may submit solutions to any or all of these problems.
3. Solutions should be handed in on or before Friday 31 January 2014 to either Dr. A. Kay (W239) or Prof. A.P. Veselov (W233ma), who will be the judges for the Challenge.