Applied Maths Seminars 2017/18 Semester 1

26 September (Tuesday)
U005 (Brockington Extension)
9:00am: Prof Al Osborne (Nonlinear Waves Research Corp / Universitá di Torino).
Integrable Turbulence for Soliton and Breather Gases Using Finite Gap Theory:
Applications to Ocean Surface and Internal Waves.
Joint seminar with SIG workshop.

Recent experimental results have verified the occurrence of soliton turbulence and breather gases in random oceanic surface waves [Costa, et al, 2014] and internal waves [Osborne, 2010]. Soliton turbulence was predicted theoretically by Zakharov [1971] many years ago as weak wave turbulence. Here I give an overview of the development of a completely integrable soliton turbulence theory for the Korteweg-deVries (KdV) and Kodomtsev-Petviashvili (KP) equations using periodic finite gap theory (FGT) [Belokolos, et al, 1994]. Likewise I develop a similar theory for the nonlinear Schrödinger (NLS) equation in 1+1 for breather turbulence. The above integrable cases are computed without resort to closure arguments, as the evolution of the power spectra are computed exactly using FGT. Extension of the method to asymptotically integrable theories for the perturbed KdV and NLS equations gives the solutions in terms of Abelian functions [Baker, 1897, 1907]. Here the closure comes with application of Lie transforms to carry the approach to asymptotically higher order [Kodama, 1985a, b] [Fokas and Liu, 1995]. A turbulent theory for the nonintegrable 2+1 NLS equation is also explored using FGT and a perturbation scheme first suggested by Poincaré [1891]. In all of the above theories the solitons and breather trains are the natural coherent structures in the integrable/quasi-integrable flows. Comparisons of the theoretical results to oceanic data sets are given. The hyperfast numerical methods for simulating the physical wave fields are based on Riemann theta functions: These are quasiperiodic Fourier series that naturally contain the coherent structures in the Riemann spectrum. Thus the analytical expressions for the time evolution of the correlation function and the power spectrum explicitly contain these coherent structures.

Costa, A. et al. Soliton turbulence in shallow water ocean surface waves. Phys. Rev. Lett. 113, 108501 (2014).
Baker, H. F., Abelian Functions: Abel's Theorem and the Allied Theory of Theta Functions [Cambridge University Press, Cambridge, 1897].
Baker, H. F., An introduction to the Theory of Multiply Periodic Functions, [Cambridge University Press, Cambridge, 1907].
Belokolos, E. D., A. I. Bobenko, V. Z. Enol'skii, A. R. Its and V. B. Matveev, Algebro-Geometric Approach to Nonlinear Integrable Equations [Springer-Verlag, Berlin, 1994].
Fokas, A. S. and Q. M. Liu, Asymptotic integrability of water waves, Phys. Rev. Lett. 77(12), 2347-2351, 1996.
Kodama, Y., Normal forms for weakly dispersive wave equations, Phys. Lett. 112A(5), 193-196, 1985a; On integrable systems with higher order corrections, Phys. Lett. 112A(6), 245-249, 1985b.
Osborne, A. R., Nonlinear Ocean Waves and the Inverse Scattering Transform [Academic Press, Boston, 2010] 976 pages.
Poincaré, H., New Methods of Celestial Mechanics, 1. Periodic and asymptotic solutions, American Institute of Physics, Ed. by Daniel L. Goroff (translation of: Les méthods

nouvelles de la méchanique céleste, originally published 1892) 1993.
Zakharov, V. E., Kinetic equation for solitons, Sov. Phys. JETP 33(3), 1971.

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4 October
SCH.1.05
2:00pm: Prof Luis G. MacDowell (Química-Física, Complutense University of Madrid)
Capillarity Theory at the Nanoscale.

In the theory of capillarity describing the shape of adsorbed fluid films, the liquid-vapor interface is characterized by a single parameter, namely, the interface tension. This yields the classical surfaces of constant curvature for the shape of the interface. For sufficiently thin films, this level of description is augmented by the introduction of an effective potential acting between the substrate and the liquid-vapor interface. This provides a partial non-linear differential equation for the equilibrium film thickness. In this talk we will discuss computer simulation experiments that probe the liquid-vapor interface of thin adsorbed films [1,2,3]. In our study, we measured the spectrum of surface fluctuations, and the results indicate that the effective surface tension that is obtained becomes dependent on the film thickness for films on the sub-nanometer scale. Such behavior can be explained in terms of a simple microscopic theory [3,4], but the implication is that the classical Young-Laplace-Derjaguin-Frumkin theory of thin adsorbed films must be corrected for films at the nanometer scale [3]. The resulting partial differential equation is not only non-linear in the film height, but also in its gradient. This finding could have implications in our understanding of line tensions, spinodal dewetting and droplet dynamics.

References:
[1] L.G. MacDowell, J. Benet and N.A. Katcho, Phys.Rev.Lett. 111 047802 (2013)
[2] L.G. MacDowell, J. Benet, N.A. Katcho and J.M.G. Palanco, Adv. Colloid Interface Sci. 206 150 (2014).
[3] J. Benet, E. Sanz and L.G. MacDowell, J.Phys.Chem. C 118 22079 (2014).
[4] L.G. MacDowell, Phys. Rev. E, 96, 022801 (2017).

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11 October
SCH.1.05
1:00pm: Dr Daniel Ratliff (Mathematical Sciences, Loughborough University)
Modulation of Multiphase Wavetrains and Nonlinear Reductions.

The modulation of single phase wavetrains (and more recently their generalisation, relative equilibria), instigated by Whitham, is a field that has been developed over the last 50 years. The procedure generates a set of dispersionless nonlinear PDEs that govern the local wavenumber and frequency of the wave. When these degenerate, it has been shown that dispersion emerges at such points leading to equations such as the Korteweg-de Vries (KdV) equation. Remarkably, such reductions possess coefficients that may be related to the conservation laws of the original system which can be calculated in advance. This property is known as ‘universal form’.

This talk concerns itself with taking these ideas and applying them to solutions that have more than one phase. The questions are now this – which nonlinear PDEs arise? Do *these nonlinear reductions still emerge with universal form? It will be shown that yes, these properties generalise quite nicely to the multiphase problem and recover many of the same equations derived from single phased solutions (like the KdV). Unsurprisingly, the increase in the number of system parameters allows one to derive further nonlinear PDEs (and even some new ones).*

The talk concludes (hopefully, time permitting) by discussing two applications of the theory. The first is a stratified shallow water system and the second is a set of coupled Nonlinear Schrodinger equations (which model ocean wave envelopes, Bose-Einstein condensates and electromagnetic waves), showing the possible reductions and how the conditions for each equation can be met.

This work is in collaboration with Tom Bridges (University of Surrey).

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18 October
SCH.1.05
2:00pm: Dr Lorna Ayton (EPSRC Early Career Research Fellow, DAMTP, University of Cambridge)
Reducing Aerofoil Noise by Redesigning Leading Edges.

Aerodynamic noise is a fundamental concern facing the aviation industry, whether it's the noise generated by a passenger aeroplane, or by a delivery drone. The key feature linking all aerodynamic designs is aerofoils/blades, and these generate both leading- and trailing-edge noise through interaction with unsteady fluid flows. This talk will first discuss the basics of noise generation by aerofoils in unsteady subsonic flows followed by discussing new bio-inspired adapted blade designs for reducing leading-edge noise. It is important for any new design that we understand how it interacts with the flow to reduce noise so that the optimal design can be used in a variety of flow regimes. This talk will therefore present mathematical models that are capable of quickly predicting the generated noise and can be used to infer the noise-reduction mechanisms of blade adaptations.

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1 November
SCH.1.05
2:00pm: Dr Shailesh Naire (School of Computing and Mathematics, Keele University)
The spreading of a surfactant-laden drop down an inclined and pre-wetted substrate - Numerics, Asymptotics and Linear Stability Analysis

Surfactants are chemicals that adsorb onto the air-liquid interface and lower the surface tension there. Non-uniformities in surfactant concentration result in surface tension gradients leading to a surface shear stress, known as a Marangoni stress. This stress, if sufficiently large, can influence the flow at the interface.

Surfactants are ubiquitous in many aspects of technology and industry to control the wetting properties of liquids due to their ability to modify surface tension. They are used in detergents, crop spraying, coating processes and oil recovery. Surfactants also occur naturally, for example in the mammalian lung. They reduce the surface tension within the liquid lining the airways, which assists in preventing the collapse of the smaller airways.

In the lungs of premature infants, the quantity of surfactant produced is insufficient as the lungs are under-developed. This leads to a respiratory distress syndrome which is treated by Surfactant Replacement Therapy.

Motivated by this medical application, we theoretically investigate a model problem involving the spreading of a drop laden with an insoluble surfactant down an inclined and pre-wetted substrate. Our focus is in understanding the mechanisms behind a “fingering” instability observed experimentally during the spreading process. High-resolution numerics reveal a multi-region asymptotic wave-like structure of the spreading droplet. Approximate solutions for each region is then derived using asymptotic analysis. In particular, a quasi-steady similarity solution is obtained for the leading edge of the droplet. A linear stability analysis of this region shows that the base state is linearly unstable to long-wavelength perturbations. The Marangoni effect is shown to be the dominant driving mechanism behind this instability at small wavenumbers. A small wavenumber stability criterion is derived and it's implication on the onset of the fingering instability will be discussed.

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8 November
SCH.1.05
2:00pm: Matt Tranter (Mathematical Sciences, Loughborough University)
Detecting delamination in layered structures with the help of solitons

In this talk we will discuss the modelling of long longitudinal bulk strain solitary waves in delaminated elastic bars. We consider both pure and radiating solitary waves. We develop direct and semi-analytical numerical approaches to the problem, using asymptotic multiple-scale expansions and averaging with respect to the fast variables. We also obtain partial theoretical estimates using the Inverse Scattering Transform. The results indicate that solitary waves can be used to detect delamination.

This is a joint work with Karima Khusnutdinova.

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15 November
SCH.1.05
2:20pm: Federica Buriani (Mathematical Sciences, Loughborough University)
Hydrodynamics of a flexible piezoelectric wave energy harvester moored on a breakwater

Piezoelectric wave energy converters (WECs) are an innovative concept of electromechanical ocean energy converters for low-power applications. In this talk, we analyse the interaction of linear water waves with a flexible piezoelectric structure submerged in water of finite depth. One end of the converter is clamped to a rigid support system in the ocean, while the other end is moored on a vertical wall, e.g. a caisson breakwater. This configuration reproduces a possible real application of piezoelectric WECs, in which the wave-structure interaction plays a fundamental role, as it determines the superimposition of incident, radiated and reflected wave components. The interaction of such components affects the power output of the device, yielding power peaks when the flexible plate resonates with the wave system.

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22 November
SCH.1.05
2:00pm: Dr Jun Zang (Director of Research Unit for Water, Environment and Infrastructure Resilience (WEIR), University of Bath)
Recent Advances in Wave-Structure Interactions – Towards Accurate and Efficient Numerical Modelling

This presentation will highlight our recent advances in the development and application of advanced numerical methods to challenging coastal and offshore problems. By implementing appropriate numerical methods, different problems, such as performance and survivability analysis of wave energy devices, wave over-topping of breakwaters, and wave impact on floating structures can be modelled more accurately and efficiently. In the presentation, I shall give an overview on our recent studies on applying novel Particle-in-Cell method, open source CFD tool OpenFOAM and traditional potential flow solver to wave interaction with fixed and floating structures.

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29 November
SCH.1.05
2:00pm: Dr Benjamin Aymard (Inria Sophia Antipolis Mediterranee Research Centre)
Dynamical phase separation using a microfluidic device: experiments and modeling

We study the dynamical phase separation of a binary fluid by a microfluidic device both from the experimental and from the modeling points of view. The experimental device consists of a main channel (600μm wide) leading into an array of 276 trapezoidal capillaries of 5μm width arranged on both sides and separating the lateral channels from the main channel. Due to geometrical effects as well as wetting properties of the substrate, and under well chosen pressure boundary conditions, a multiphase flow introduced into the main channel gets separated at the capillaries. Understanding this dynamics via modeling and numerical simulation is a crucial step in designing future efficient micro-separators. We propose a diffuse-interface model, based on the classical Cahn-Hilliard-Navier-Stokes system, with a new nonlinear mobility and new wetting boundary conditions. We also propose a novel numerical method using a finite-element approach, together with an adaptive mesh refinement strategy. The complex geometry is captured using the same computer-aided design files as the ones adopted in the fabrication of the actual device. Numerical simulations reveal a very good qualitative agreement between model and experiments, demonstrating also a clear separation of phases.

This work is in collaboration with Urbain Vaes, Imperial College London, Anand Radhakrishnan, University College London, Marc Pradas, The Open University, Asterios Gavriilidis, University College London, Serafim Kalliadasis, Imperial College London.

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4 December - Note different date/time/room
SCH.0.01
4:00pm: Dr Hayder Salman (School of Mathematics, UEA)
Long-range Ordering and Negative Temperature States of Quantized Vortices in a Two-

Dimensional Superfluid

We study the relaxation of a 2D superfluid from a nonequilibrium initial state consisting of vortices with positive and negative circulation in experimentally realizable square and rectangular traps. We focus on how like-signed quantized vortices can form clusters and show that such clustering can be understood in terms of negative temperature states of a vortex gas. Using a mean field approximation for the vortex gas, we identify an order parameter that is related to the formation of long-range correlations between the vortices. It turns out that the order parameter corresponds to the streamfunction of a 2D flow field that is governed by a Boltzmann-Poisson equation. It is, therefore, associated with the emergence of a mean rotational hydrodynamic flow with a nonzero coarse-grained vorticity field. Solutions of the Boltzmann-Poisson equation in a square domain reveal that maximum entropy states of the vortex gas correspond to a large scale monopole flow field. A striking feature of this mean flow, is the spontaneous acquisition of angular momentum by a superfluid flow with a neutral vortex charge. These mean-field predictions are verified through direct simulations of a point vortex gas and 2D simulations of the Gross-Pitaevskii equation. Due to the long-range nature of the Coulomb-like interactions in point vortex flows, the negative temperature states strongly depend on the shape of the geometry. By modifying the domain to a rectangular region, we identify a geometry induced phase transition of the most probable mean flow field which our numerical simulations reproduce.

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12 December - Note different date/room
SCH.1.01
2:00pm: José C. B. da Silva (Faculdade de Ciencias da Universidade do Porto, Porto, Portugal)
Direct resonance between multimodal internal waves in the ocean: SAR observations and Modelling

Direct resonance between multimodal internal waves in the ocean is discussed based on high resolution satellite observations. In particular we investigate the generation of Internal Solitary Waves (ISWs) at work to the east of the Mascarene Plateau (Indian Ocean) using Synthetic Aperture Radar (SAR) imagery and MITgcm fully nonlinear and nonhydrostatic simulations. Realistic representations of stratification and bathymetry are used with asymmetric tidal forcing (including the steady South Equatorial Current which is assumed barotropic in the model) along a 2D transect aligned with the propagation direction of the wave signatures identified in the SAR. We focus on mode-2 ISW-like waves whose length-scales are O(20km) appear some 50 km upstream of the sill, after an Internal Tide (IT) beam scatters into the pycnocline, itself originating from critical topography on the leeward (i.e. westward) side of the sill. The large-scale mode-2 ISW-like waves that form far upstream from the sill are long-lived features and can be identified in the SAR due to associated short-scale mode-1 ISWs which propagate with the same phase speed, i.e. in resonance. This coupling is also seen in the model, and here it is argued that the formation of those mode-2 ISW-like waves appears to originate from the IT beam after it reflects from the sea surface and interacts with the pycnocline. The resonant coupled waves and this IW generation process may be easily overlooked and could be at work in many more regions of the world than previously thought. Here we also provide new evidence of SAR images that are consistent with mode-2 solitary-like IWs with scales of less than one km in the Andaman Sea. Linear theory suggests resonant coupling with long internal tides (internal waves of tidal frequency) with higher-mode vertical structures (mode-4), possibly originating from the steep slopes along the Ten degree Channel of the Andaman Sea.

Seminar name: Applied Maths Seminar

Year: 2017/18

Semester 2

Date	Time/place	Speaker	Title
31st January 2018	14:00-15:00/SCH.0.01 Schofield Building	Prof Mark Hoefer (Applied Mathematics, University of Colorado Boulder)	Solitary Wave-Mean Flow Interaction -or- Surfing Solitons
31st January 2018	15:00-16:00/SCH.0.01 Schofield Building	Dr Christian Buckingham (British Antarctic Survey)	Examining mechanisms of submesoscale eddy generation in the open ocean using observations
8th February 2018	15:00-16:00/U.0.06 Brockington Building	Dr Matthew Scase (University of Nottingham)	Rotating Rayleigh-Taylor Instability
15th February 2018	14:00-15:00/SCH.1.05 Schofield Building	Dr Kenny Jolley	Atomistic modelling of defects in graphite
21st February 2018	14:00-15:00/SCH.0.01 Schofield Building	Dr Gyula Tóth (Loughborough University)	Phase-field modelling of pattern formation in quasi-incompressible multicomponent liquids
28th February 2018	14:00-15:00/SCH.0.01 Schofield Building	Dr Thibault Congy (Loughborough University)	Riemann problem for polarization waves in a two-component Bose-Einstein condensate
11th April 2018	14:00-15:00/SCH.0.01 Schofield Building	Dr. Lyuba Chumakova (The University of Edinburgh)	Leaky dispersive waves
12th April 2018	14:00-15:00/SCH.0.01 Schofield Building	Dr. Radu Cimpeanu (University of Oxford)	How to make a splash: high speed drop impact in aerodynamics
18th April 2018	14:00-15:00/SCH.0.01 Schofield Building	Prof. Sergey Gavrilyuk (Université Aix-Marseille, France)	Multi-dimensional shear shallow water flows
25th April 2018	14:00-15:00/SCH.0.01 Schofield Building	Dr. Danila Prikazchikov (Keele University)	Low-frequency spectra of high-contrast composite structures
2nd May 2018	14:00-15:00/ WAV.0.37 Wavy Top Building	Dr Jennifer Creaser (Exeter)	Sequential escapes for network dynamics
17th May 2018	14:00-15:00/SCH.1.01 Schofield Building	Dr. Tamás Pusztai (Wigner Research	Topological defects in 2D orientation-field

		Centre for Physics, Hungary)	based phase-field models
30th May 2018	14:00-15:00/SCH.1.01 Schofield Building	Dr. Miguel A. Alejo (Federal University of Santa Caterina, Brazil)	On the stability properties of some breather solutions
30th May 2018	15:00-16:00/SCH.1.01 Schofield Building	Dr. Svetlana Gurevich (Institute for Theoretical Physics, University of Münster, Germany)	Delayed feedback control of self-mobile cavity solitons in a wide-aperture laser with a saturable absorber
6th June 2018	14:00-15:00/SCH.1.01 Schofield Building	Prof Miguel Bustamante (University College Dublin, Ireland).	Energy flux enhancement, intermittency and turbulence via Fourier triad phase dynamics in 1D Burgers equation